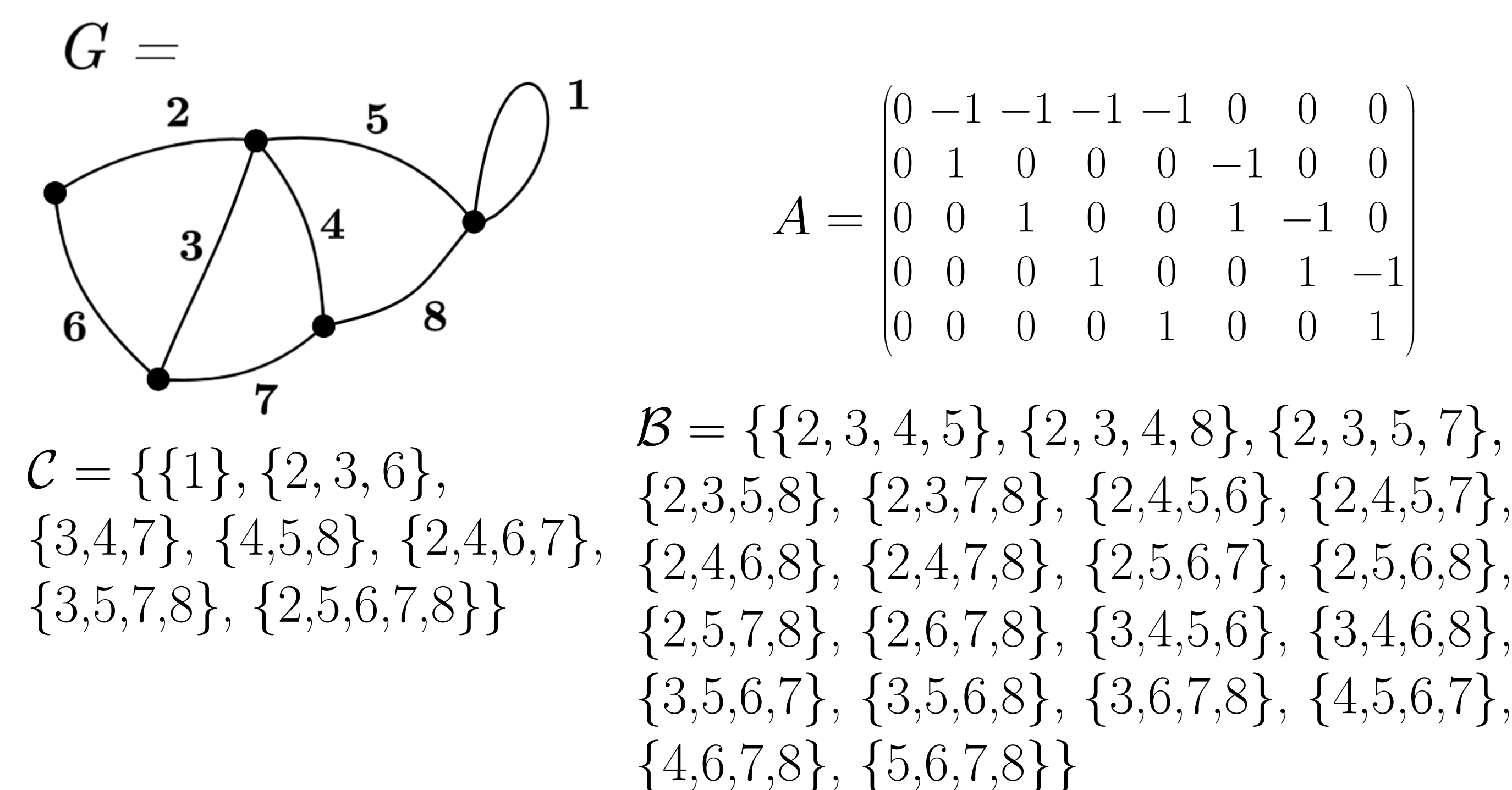


What Is a Matroid?

A **matroid** $M = (E, \mathcal{I})$ consists of a finite set E and a collection $\mathcal{I}$ of subsets of E called **independent sets**. Independent sets satisfy the following axioms:

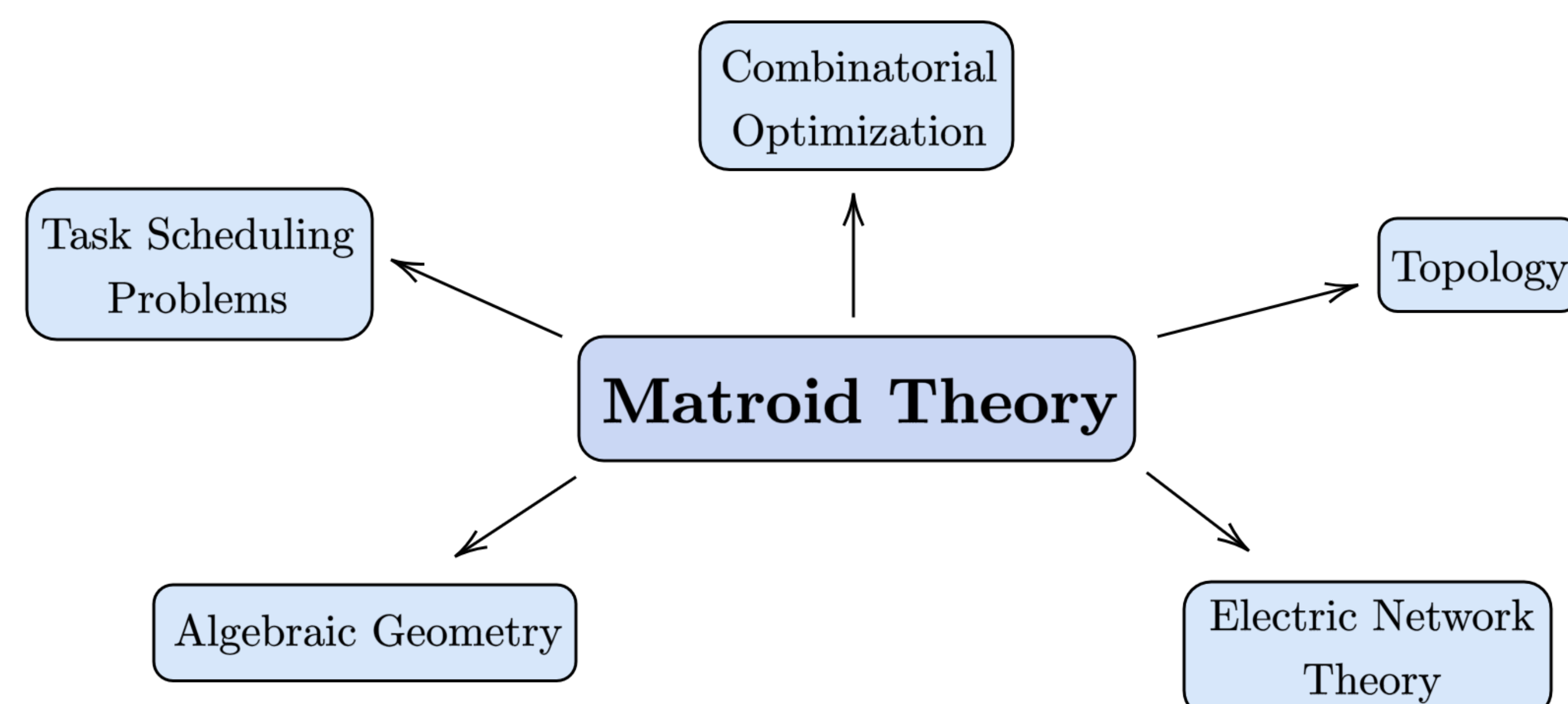
- (i) $\emptyset \in \mathcal{I}$
- (ii) If $I_1 \in \mathcal{I}$ and $I_2 \subseteq I_1$, then $I_2 \in \mathcal{I}$
- (iii) If $I_1, I_2 \in \mathcal{I}$ and $|I_2| > |I_1|$, then there exists $e \in I_2 - I_1$ such that $I_1 \cup \{e\} \in \mathcal{I}$

A matroid can also be defined by its *bases*, *circuits*, *dependent sets*, *rank function*, *flats*, and *hyperplanes*. They may also be constructed from mathematical objects:



Matroid Uses and Applications

Matroids abstract the notion of independence, a fundamental mathematical concept. Consequently, they arise all over mathematics and science.

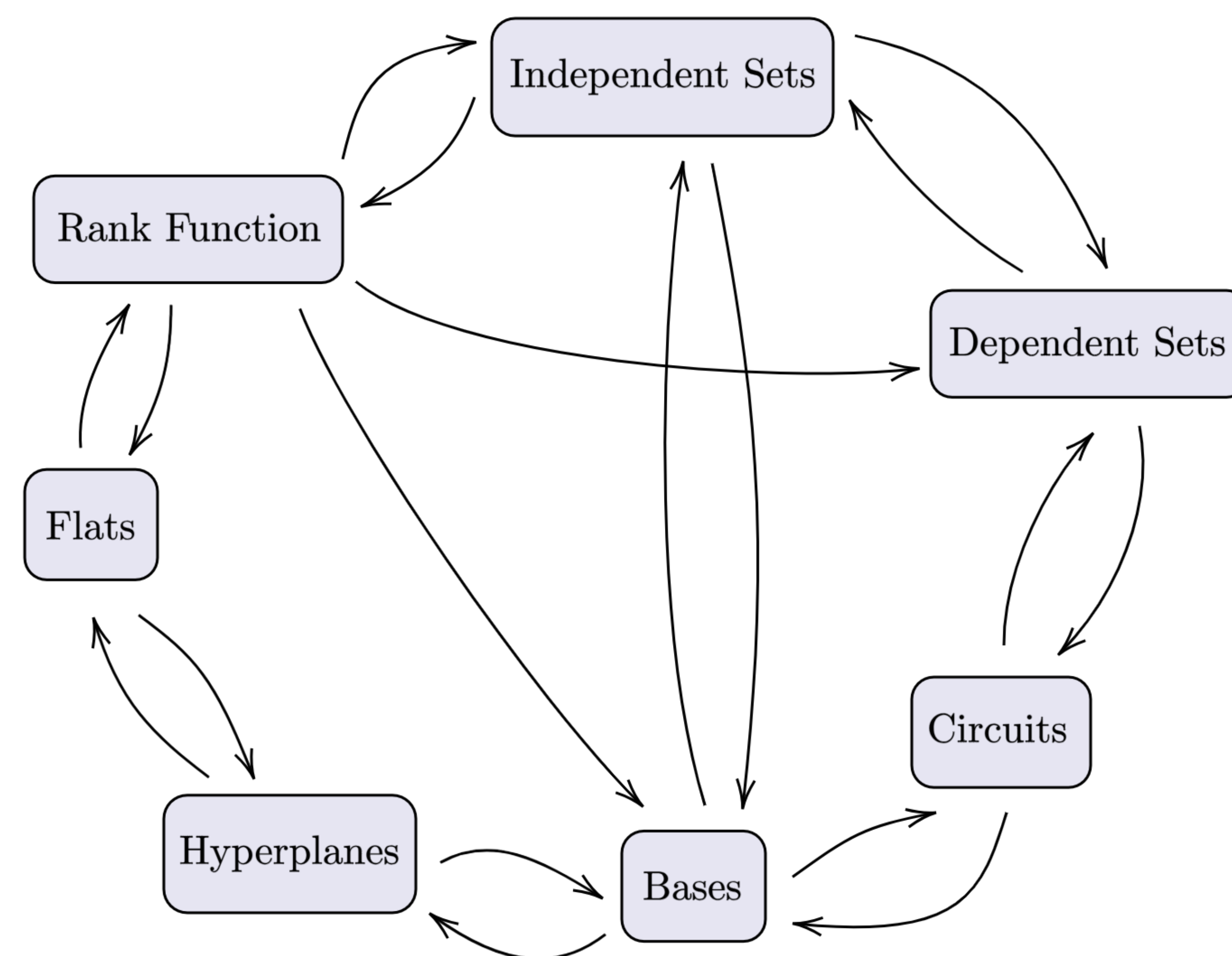


Features and Functionality

The **Matroids** package in **Maple** is designed with efficiency in mind. A matroid in **Maple** may be created from axioms, or from objects like graphs, matrices, and polynomials ideals. Once created, the user can use the **Matroids** functions to quickly compute any of its several representations, the Tutte polynomial, its dual, rank and more.

Cryptomorphisms

A **cryptomorphism** is an algorithm that converts one representation of an object into another. The illustration below describes the matroid cryptomorphisms implemented in the **Matroids** package.



Matroid cryptomorphisms are famously expensive. We have implemented several strategies to mitigate their cost:

- We encode sets of sets as **hypergraphs** from the **Hypergraphs** package in **Maple**
- In **Hypergraphs**, sets are encoded as **bit vectors**
- We follow a **lazy evaluation** approach to computation

Maple Worksheet Demonstration

```
> M := Matroid(A);
      0  -1  -1  -1  -1  0  0  0
      0  1   0   0   0  -1  0  0
      0  0   1   0   0   1  -1  0
      0  0   0   1   0   0   1  -1
      0  0   0   0   1   0   0   1
M :=
"The linear matroid whose ground set is the set of column vectors of the
above matrix"
> Bases(M);
[[{2, 3, 4, 5}, {2, 4, 5, 6}, {3, 4, 5, 6}, {2, 3, 5, 7}, {2, 4, 5, 7}, {2, 5, 6, 7}, {3, 5, 6, 7}, {4, 5, 6, 7}, {2, 3, 4, 8}, {2, 3, 5, 8}, {2, 4, 6, 8}, {3, 4, 6, 8}, {2, 5, 6, 8}, {3, 5, 6, 8}, {2, 3, 7, 8}, {2, 4, 7, 8}, {2, 5, 7, 8}, {2, 6, 7, 8}, {3, 6, 7, 8}, {4, 6, 7, 8}, {5, 6, 7, 8}]
> Hyperplanes(M);
[[{1, 2, 5, 7}, {1, 5, 6, 7}, {1, 2, 7, 8}, {1, 6, 7, 8}, {1, 2, 3, 5, 6}, {1, 2, 4, 5, 8}, {1, 2, 3, 6, 8}, {1, 4, 5, 6, 8}, {1, 2, 3, 4, 6, 7}, {1, 3, 4, 5, 7, 8}]
> TuttePolynomial(M);
x^4 y + 3 x^3 y + 3 x^2 y^2 + 2 x y^3 + y^4 + 3 x^2 y + 4 x y^2 + 2 y^3 + x y + y^2
> Dual(M);
"A matroid on 8 elements with 21 bases of size 4"
> N := Matroid(G);
      1  2  3  4  5  6  7  8
      1  5  8  4  3  7  2  6
N :=
"The graphic matroid on the above graph"
> Circuits(N);
[[{1}, {2, 3, 6}, {3, 4, 7}, {4, 5, 8}, {2, 4, 6, 7}, {3, 5, 7, 8}, {2, 5, 6, 7, 8}]
> AreIsomorphic(N,M);
true
```

Acknowledgements

We thank our supervisors, Taylor Brysiewicz and Marc Moreno Maza, for their guidance, insight, and support, as well as Erik Postma. We are also grateful for the provisions of the ORCCA lab. This work was supported by funding from the USRI program.

References

- [1] Maplesoft.
Maple (2023).
Maplesoft: A division of Waterloo Inc., Waterloo, Ontario.
- [2] J. Oxley.
Matroid Theory.
Oxford graduate texts in mathematics. Oxford University Press, 2006.