CS490y Undergraduate Thesis Presentation

Organizing the Structure of Mathematical Expressions

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mathml @ORCCA

What is the problem?

- Re-associate symbols and characters in *single-line* mathematical expressions while preserving expressions' implicit semantics
- Generate *Presentation MathML* as the result of the analysis

Keywords: MathML, Mathematical Handwriting Recognition

Why is this problem interesting?

- Need to recognize and treat mathematical expressions by computers in a meaningful manner
  - Mathematical handwriting recognizer
  - TEX/MathML converter
- Capture the semantics of mathematics so that the mathematical expressions can be:
  - Computed using a Computer Algebra System
  - Stored in databases

An example of (bad) character re-association

- A TEX markup of $\int_a^b (x+3)^2 dx$:

```
$ \int_{a}^{b}\! {(x+3} {)}^{2} {dx} $
```

- Curly brackets indicate implicit groupings in TEX
  - Do we actually mean to apply the exponent to ")" in "`{)}^{2}`"?
  - Do we actually mean to group "$(x+3$" together in "`{(x+3}`"?
  - etc.

We can see...

- If we input math by handwriting
  - How to associate characters and symbols using two-dimensional information provided?
  - How to make handwriting recognizers to recognize two-dimensional data?
  - A picture of an expression does not capture the semantics!
  - ...

How can one extract the semantics of mathematics from the representation of the expressions?

Previous works

- In University of Western Ontario
  - Bo Wan, a former member of ORCCA, developed a mathematical handwriting recognizer for the pocket PC
- In Université de Nice, Sophia-Antipolis, France
  - Stéphane Lavirotte developed an OFR (Optical Formula Recognition) to recognize mathematics in documents
  - A set of *graph grammars* defines permitted two-dimensional relationships between characters and symbols in mathematical expressions

Both of these works contain similar discussions

A brief overview of MathML

- A W3C recommendation to put mathematics on the web
- Looks a bit like HTML
- XML (eXtensible Markup Language)
- Natively supported by Netscape 7 and Mozilla
- Two kinds of markup:

  Presentation MathML encodes how the mathematical expressions look

  Content MathML encodes the semantics and the meaning of mathematical expressions

- We work with *Presentation MathML* in this project

- Example: MathML markup for "x^3":
  - Content MathML

```
<math xmlns="http://www.w3.org/MathML">
   <apply>
      <exp/>
      <ci> x </ci>
      <cn> 3 </cn>
   </apply>
</math>
```

  - Presentation MathML

```
<math xmlns="http://www.w3.org/MathML">
   <mrow>
      <msup>
         <mi> x </mi>
         <mn> 3 </mn>
      </msup>
   </mrow>
</math>
```

Properties of mathematical expressions

- *Presentation* VS *Content*
  - Presentation
    - concerns how the expressions look
  - Content
    - concerns the semantics of expressions

We intuitively draw the relationships between the presentation and the content of mathematics

- Example: "x^3" written in TeX, and OpenMath:
  - TeX:

```
$ {x}_{}^{3} $
```

  - OpenMath:

```
<OMOBJ>
   <OMA>
      <OMS cd="tranc1" name="exp"/>
      <OMV name="x"/>
      <OMI> 3 </OMI>
   </OMA>
</OMOBJ>
```

- Two-Dimensional
  - How can we know that "x^3" does not equals to "$x3$"?
  - Two-dimensional relationships are vital to determine the content of mathematical expressions
  - The relationships are:
    - *Subscript* (Example: x_y)
    - *Superscript* (Example: x^y)
    - *Underscript* (Example: $\underset{\cdot}{x}$)
    - *Overscript* (Example: $\dot{x}$)
    - *Presuperscript* (Example: ${}^yF$)
    - *Presubscript* (Example: ${}_yF$)
    - *Inline* (Example: xy)
    - *Include* (Example: $\sqrt{x}$)

- Uses of notations are arbitrary
  - Precedence
    - **"BEDMAS"**
      - **B**racket
      - **E**xponent
      - **D**ivision
      - **M**ultiplication
      - **A**ddition
      - **S**ubtraction
  - Number of arguments
    - Unary, binary, and n-ary
  - Some of the arguments are compulsory
    - Why $\int(x+3)^2dx$ is valid and $\int_0(x+3)^2dx$ is not?

- Location of Operators
  - *Prefix* (Example: $-x$)
  - *Infix* (Example: $1+2+3$)
  - *Postfix* (Example: $3!$)
  - *Bounding* (Example: $[a, b]$)
  - *Implicit* (Example: x^y)
  - *Two-dimensional* (Example: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$)
  - *Include* (Example: $\sqrt{x}$)

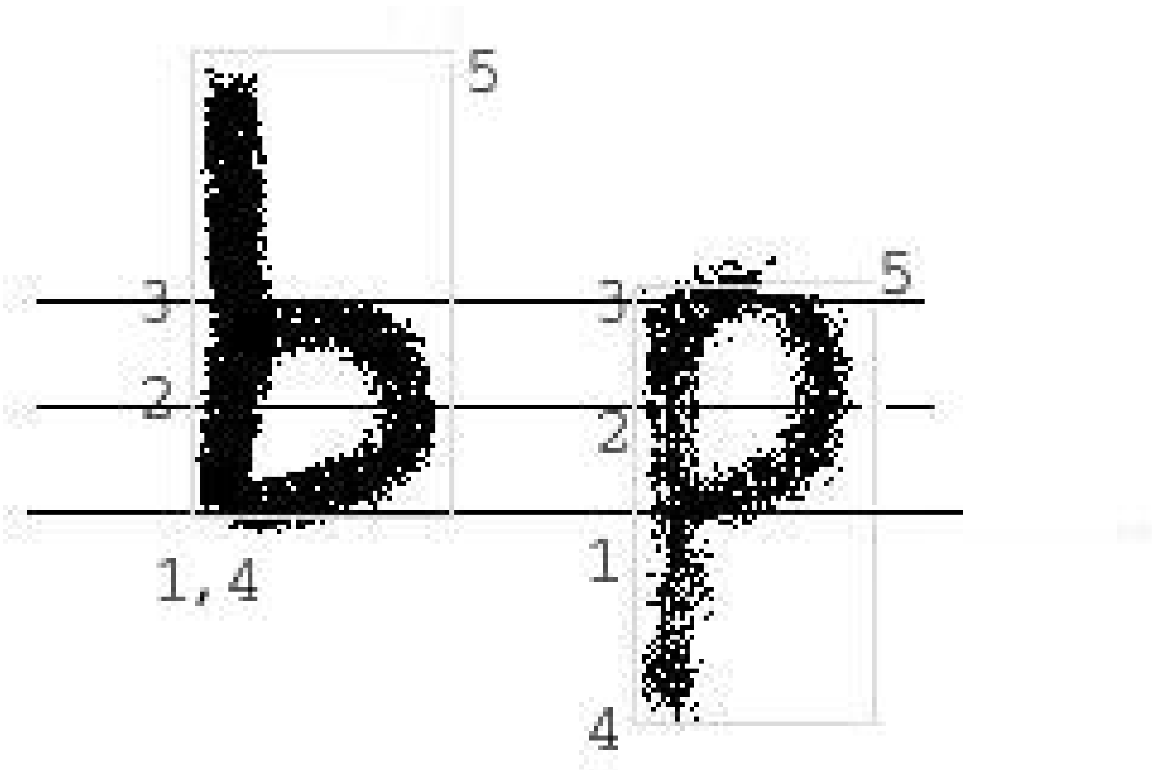
- Meanings must be determined globally
  - Example: a dot in "3.5", "$3 \cdot 5$" and "$\ldots$"

- Variations and ambiguities in notations
  - Variations
    - Example: decimal point
      - English: 3.5
      - French: $3,5$
  - Ambiguities
    - Example: Does $\sum_{x=1}^{10} x + 1$ mean
      "$1 + 2 + 3 \ \ldots + 10 \ + 1$"
      or
      "$(1 + 1) + (2 + 1) + (3 + 1) + \cdots + (10 + 1)$"?

Recognizing the semantics of mathematical expressions is very complex

Requirements for recognizing mathematical expressions

- Every character is enclosed by a *bounding box*



- Information provided by a *bounding box*:
  1. Absolute reference point
  2. Body midline
  3. Body topline
  4. Lower left corner of the *bounding box*
  5. Upper right corner of the *bounding box*

- Why having the "body" lines?
  - To determine superscript relationship
  - Superscripts are written relative to the "body" lines
  - We understand "2" is the superscript of in:

$$b^2 \qquad p^2 \qquad a^2$$

- The order of the characters in the inputs must be in a certain order
  - To reduce the size of the problem

- Only **single-line** mathematical expressions is covered in this project:
  - No array, fractions, or table constructs allowed
  - For example, $|x| = \begin{cases} -x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ x & \text{if } x > 0 \end{cases}$ and $\begin{bmatrix} x^2+1 & x-1 \\ x-1 & 1 \end{bmatrix}$ are obviously not single-line mathematical expressions

Structual Analysis and Presentation MathML Generation

- Remember:
  - A piece of MathML markup is a n-way tree
  - Every node of the tree is a tag (Example: `<math>`)
  - Every tag *may* contain text
  - *Offline* process is assumed for this project
  - We get the characters and their associated bounding boxes in a input file, one expression per file

- Organizing characters in mathematical expression take five steps:
  1. Generate “flat” Presentation MathML
  2. Merge digits and predefined character sequences
  3. Recognize two-dimensional relationships
  4. Replace parenthesis with `<mfenced>`
  5. Indicate implicit grouping by adding `<mrow>`

Idea:
Put all symbols and characters in an internal tree
and re-arrange the tree nodes
as new relationships are identified

An example of organizing characters in an expression

Let's try to do $\int_0^\infty \sin^{12} x \, dx$

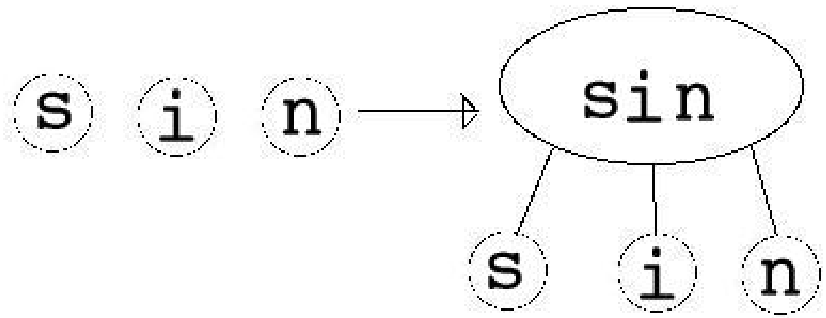
1. Generate "flat" MathML
   - Assume that all individual characters are in the same row
   - Put the characters in different categories:
     - number → `<mn>`
     - identifier → `<mi>`
     - operators → `<mo>`

- "Flat" MathML of $\int_0^\infty \sin^{12} x \; dx$:

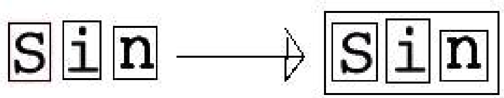
```
<math xmlns="http://www.w3.org/MathML/">
  <mrow>
    <mo> &int; </mo>
    <mi> &infin; </mi>
    <mn> 0 </mn>
    <mi> s </mi>
    <mi> i </mi>
    <mi> n </mi>
    <mn> 1 </mn>
    <mn> 2 </mn>
    <mi> x </mi>
    <mi> d </mi>
    <mi> x </mi>
  </mrow>
</math>
```

2. Merge digits and predefined character sequences

- In $\int_0^\infty \sin^{12} x \, dx$:
  - "1" and "2" can be grouped together to form "12"
  - "s", "i", and "n" together formed "sin"
  - A new node is created to store the merged characters:



  - A new *bounding box* is created upon the creation of a new node:



- Merged digits are still <mn> (a number)
- Merged character sequences are changed to <mo> (an operator)
- MathML generated so far for $\int_0^\infty \sin^{12} x$:

```
<math xmlns="http://www.w3.org/MathML/">
   <mrow>
      <mo> &int; </mo>
      <mi> &infin; </mi>
      <mn> 0 </mn>
      <mo> sin </mo>
      <mn> 12 </mn>
      <mi> x </mi>
      <mo> dx </mo>
   </mrow>
 </math>
```

3. Recognize two-dimensional relationships (Partially implemented)

- In $\int_0^\infty \sin^{12} x \, dx$:
  - "12" is the superscript of "sin"
  - Make a new node ("$<\texttt{msup}>$") to indicate the relationship
  - "12" and "sin" become the children of the new node
  - A new *bounding box* is created to surround the children
  - We do the same for all superscripts and subscripts

– Expected MathML generated so far:

```
<math xmlns="http://www.w3.org/MathML/">
   <mrow>
      <munderover>
         <mo> &int; </mo>
         <mn> 0 </mn>
         <mi> &infin; </mi>
      </munderover>
      <msup>
         <mo> sin </mo>
         <mn> 12 </mn>
      </msup>
      <mi> x </mi>
      <mo> dx </mo>
   </mrow>
</math>
```

4. Replace parenthesis with `<mfenced>`
   (Partially implemented)

   - The MathML should be rendered correctly before this step
   - Making sure that parenthesis are well nested

5. Indicate implicit groups by adding `<mrow>`
   (not implemented)

   - Just like "{" and "}" in LaTeX
   - Need to write rules to group characters and symbols together

Future works

- Better method to tolerate noise in input data
- Better approaches to determine the two-dimensional relationships between characters and symbols
  - Over and superscript
  - Under and subscript
- Insert more semantic information in MathML
  - Collect more predefined character sequences
  - Insert `<mrow>`s intelligently

Conclusion

- Recognizing and preserving semantics of mathematical expressions is not easy
- We try to reduce the complexities of the problem by:
  - defining information fetched from each *bounding boxes*
  - suggesting the steps to associate symbols and characters

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References

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