Compact Representation and Recognition for Handwritten Mathematical Characters

Stephen M. Watt

Ontario Research Centre for Computer Algebra

University of Western Ontario

London Ontario, Canada

Joint work with Oleg Golubitsky

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What this talk is about

- The main problem:
  In handwriting recognition, the human and the computer take turns thinking and sitting idle. Small devices therefore challenged by MHR.

- We ask:
  Can the computer do useful work while the user is writing and thereby get the answer faster after the user stops writing?

- We show:
  The answer is "Yes"!

Outline

- Mathematical handwriting recognition
- Representing curve traces using truncated orthogonal series
- A complexity model for on-line computation
- An on-line algorithm to compute series coefficients

Math Handwriting Recognition

- Considered natural and desirable by many

- Different than natural language recog:
  - 2-D layout is similar to a combination of writing and drawing.
  - No fixed dictionary.
  - Many more similar few-stroke characters.

Usual Character Recog Methods

- Smooth and then re-sample data

- Match against *N* models,   *OR....*

- Identify "features", such as
  - Coordinate values of sample points, Number of loops, cusps, Writing direction at selected points, Center of mass, *etc*

  Use a classification method, such as
  - Nearest neighbour, Subspace projection, Cluster analysis, Support Vector Machines

- Rank choices by consulting dictionary

Difficulties

- Many characters in mathematics means comparison against all models slow.
- Determining features from points
  - Requires many *ad hoc* parameters.
  - Replaces measured points with interpolations
  - It is not clear how many points to keep, and most work depends on number of points

Series Representation

Reference Char and Watt [ICDAR 2007]

- **Main idea**: represent *x* and *y* coordinate curves as truncated series.
- **Advantages**:
  - *Compact* – few coefficients needed
  - *Geometric* – the truncation order is a property of the character set, not the device
  - *Algebraic* – desired properties of curves can be computed algebraically
  - *Numerically stable*

Series Representation – Details

- Functional inner product:

$$\langle f, g \rangle = \int_a^b f(t) g(t) w(t) \mathrm{d}t$$

- Given *a, b, w*, the basis functions can be obtained from monomials by Gram Schmidt orthogonalization and can write:

$$f(t) = \sum_{i=0}^{\infty} c_i b_i(t), \quad c_i \in \Re, b_i \in B$$

Series Representation – Details

- Coefficients may be obtained by:

$$c_i = \langle f, b_i \rangle / \langle b_i, b_i \rangle, \quad i \in \mathbb{N}_0$$

- Choosing weight fn $w(t) = 1/\sqrt{1-t^2}$ gives Chebyshev polynomial basis.

$$T_n(t) = \cos(n \arccos t)$$

- Series may be truncated, leaving a residual error that decreases with order.

Advantages of Series Rep.

- The representation is compact
- Device and scale invariance
- Other features can be computed
- Degree determined intuitively (# cusps and turns)

Disadvantages of Series Rep.

- All the work to compute the series coefficients must be done after the series is written [non-linearity of $w(t)$].
- Many other recognition techniques share this problem.
- Can we somehow be more efficient by computing the series coefficients as the data is collected?

An On-Line Complexity Model

- Input is a sequence of *N* values received at a uniform rate.
- Characterize an algorithm by
  - $T_\Delta(n)$ no. operations as *n*-th input is seen
  - $T_F(n)$ no. operations after last input is seen
- Write on-line time complexity as

$$\mathrm{OL}_n[T_\Delta(n), T_F(n)]$$

- E.g., linear insertion requires time

$$\mathrm{OL}_n[O(n), 0]$$

On-Line Complexity Model (2)

- An algorithm that takes on-line time

$$\mathrm{OL}_n[T_\Delta(n), T_F(n)]$$

takes total time

$$\sum_{i=0}^{N} T_\Delta(i) + T_F(N)$$

- That is, a factor of N can come for free.

On-Line Algorithm for Series Coeffs

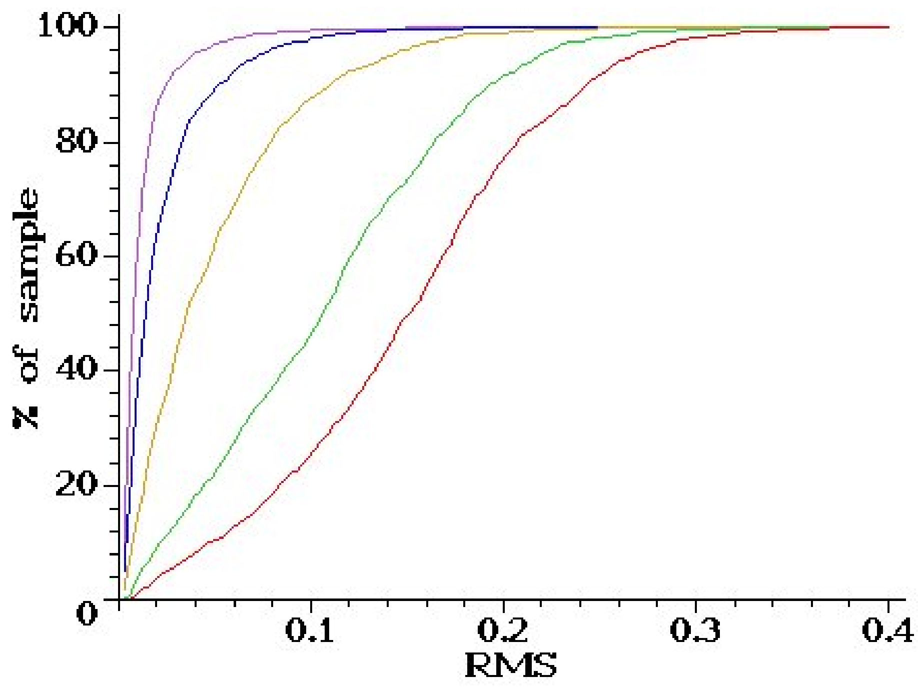
- If we choose a series whose weight function is linear, then the series coefficients can be computed on line.
- The series coefficients are linear combinations of the moments, which can be computed by numerical integration as the points are received.
- This is the Hausdorff moment problem (1921) and has been shown to be unstable by Talenti (1987).
- It is just fine, however, for the orders we need.

On-line Series Coeffs – Outline

- Use Legendre polynomials P_i as basis on the interval $[-1,1]$, with weight function 1.
- Collect numerical values for $f(\lambda)$ on $[0, L]$. L is not known until the pen is lifted.
- As the numerical values are collected, compute the moments $\int \lambda^i f(\lambda)\, \mathrm{d}\lambda$.
- After last point, compute series coeffs for f with domain and range scaled to $[-1,1]$. These will be linear combinations of he moments.

Quality of Legendre Series

- Legendre series give the same order of RMS error as the Chebyshev series representation, but can be computed on-line.



On-line Series Coeffs – Details

$$\hat{f}(\tau) = f\big((\tau+1)L/2\big) = \textstyle\sum_{n=0}^{\infty} \hat{\alpha}_n P_n(\tau)$$

$$\begin{aligned}
\hat{\alpha}_n &= (n+\tfrac{1}{2}) \int_{-1}^{1} \hat{f}(\tau) P_n(\tau)\, \mathrm{d}\tau \\
&= \frac{2n+1}{L} \int_{0}^{L} f(\lambda) P_n(2\lambda/L - 1)\, \mathrm{d}\lambda \\
&= \frac{2n+1}{L} \sum_{i=0}^{n} [t^i] P_n(2t-1) \int_{0}^{L} f(\lambda)\, (\lambda/L)^i\, \mathrm{d}\lambda \\
&= \frac{2n+1}{L} \sum_{i=0}^{n} \frac{[t^i] P_n(2t-1)}{L^i} \mu_i(f, L).
\end{aligned}$$

$$\hat{\alpha}_n = (-1)^n \frac{2n+1}{L} \sum_{i=0}^{n} \left(\frac{-1}{L}\right)^i \binom{n}{i} \binom{n+i}{i} \mu_i(f, L)$$

On-line Series Coeffs – Details

- A specialized numerical integration scheme is required for the moments.

$$\int_0^L \lambda^n f(\lambda) d\lambda \approx$$

$$\sum_{i=1}^{L-1} \frac{i^{n+1}}{n+1} \times \frac{f(i-1) - f(i+1)}{2} +$$

$$\frac{L^{n+1}}{n+1} \times \frac{f(L-1) + f(L)}{2}.$$

On-line Series Coeffs – Details

- The range of f can be scaled to any desired range $[a, b]$

$$\hat{\hat{f}}(\tau) = \frac{b-a}{f_{\max} - f_{\min}} \hat{f}(\tau) + \frac{a f_{\max} - b f_{\min}}{f_{\max} - f_{\min}}$$

$$= \sum_{i=0}^{\infty} \hat{\hat{\alpha}}_i P_i(\tau),$$

Complexity

- The on-line time complexity to compute coefficients for a Legendre series truncated to degree d is then

$$T_\Delta = 2(d+2)$$

$$T_F = \frac{3}{2}d^2 + \frac{11}{2}d + 10$$

- That is, the time at pen up is *constant* with respect to the number of points.

Practical Cost

- The construction of the series coefficients at the end is the real time limiting operation.
- This should take on the order of 200 to 500 machine instructions for a series of order 10 to 15.

Comparison with Models:
Distance between curves

- Some classification methods compute the distance between the input curve and models.
  E.g. Elastic matching, which takes time quadratic in the number of sample points.
- With orthogonal series representation, we have a much less expensive comparison:

$$\rho^2(C,\bar{C}) = \int_0^1 \left[x\left(\frac{t}{N}\right) - \bar{x}\left(\frac{t}{\bar{N}}\right)\right]^2 + \left[y\left(\frac{t}{N}\right) - \bar{y}\left(\frac{t}{\bar{N}}\right)\right]^2 dt$$

$$\approx \sum_{i=0}^{d} [c_i^x - c_i^{\bar{x}}]^2 + [c_i^y - c_i^{\bar{y}}]^2$$

- Linear in d. About 60-100 machine instructions!

Conclusions

- It is possible to compute series representation of characters quickly and compactly.
- This representation is compact, high-fidelity, device independent, numerically robust and allows algebraic treatment of the curves.
- The work to compute this rep. at pen-up is minimal, on the order of a few hundred machine instructions.
- Likewise, the cost to compute the distance between two curves is on the order of 100 machine instructions.
- The computations are straightforward and are suitable for hardware implementation.

References

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