ORCCA

Ontario Research Centre for Computer Algebra

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Notation Selection in Mathematical Computing Environments

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▪ Simple test 1

How do we write the following?

1. the open interval with end points a and b:

2. the base 10 logarithm of x:

3. the inner product of p and q:

4. the arctangent of 2:

▪ Simple test 1

How do we write the following?

1. the open interval with end points *a* and *b*: $]a, b[$
2. the base 10 logarithm of x: $\lg x$
3. the inner product of *p* and *q*: $p \llcorner q$
4. the arctangent of 2: $\mathrm{tg}^{-1} 2$

▪ Simple test 2

What does it mean for you?

1. lg x

2. $\tan^{-1} a$

3. $\left(\frac{p}{q}\right)$

4. u'

▪ Simple test 2

What does it mean for you?

1. lg x
   - ○ $\log_{10} x$
   - ○ $\log_2 x$

2. $\tan^{-1} a$
   - ○ cotangent a
   - ○ arctangent a

3. $\left(\frac{p}{q}\right)$
   - ○ Legendre symbol
   - ○ Parenthesized fraction

4. u'
   - ○ derivative of u
   - ○ u minutes
   - ○ transformation performed on an original u

▪ Origin of notational ambiguity

- **Mathematical context**
  - an ordinary derivative can be denoted as

    f', f_x, Df , $\frac{df}{dx}$ etc.

- **Area of application**
  - *i* for $\sqrt{-1}$ (complex analysis) vs.

    j (electrical engineering).
  - the integral

    $\int f(x)\mathrm{d}x$ (pure math) vs. $\int \mathrm{d}x\, f(x)$ (physics)

▪ Origin of notational ambiguity (cont-d)

- **National and cultural conventions**
  - the tangent function:

    tan (England, Canada) vs. *tg* (Russia, China);

  - the open interval:

    (*a, b*) (United States) vs.]*a, b*[(France).

- **Historical period**
  - e.g. ancient $3\overline{a+b}$ vs. the modern 3(*a* + *b*).

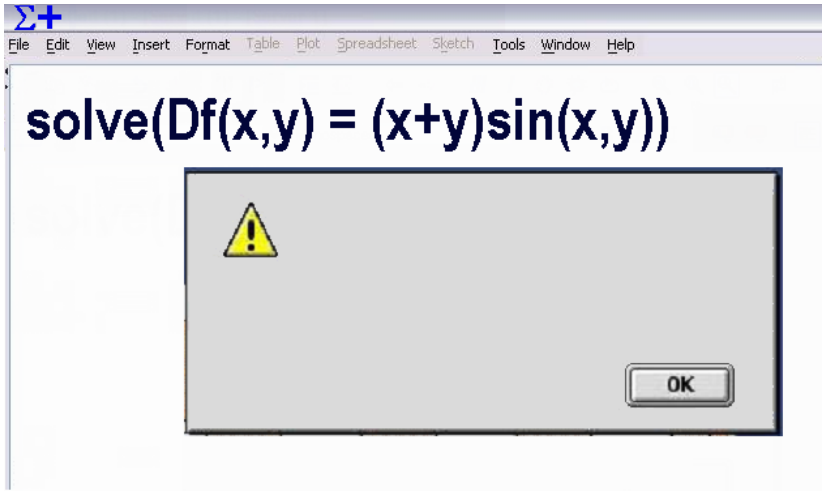
- **Level of mathematical sophistication**
  - e.g. $a \div b$ vs. $b\overline{)a}$ vs. $\frac{a}{b}$ vs. *a/b* vs. a/b.

▪ Who needs to understand written math?

- Human readers
- Software tools:
  - CAS,
  - theorem provers,
  - format converters,
  - pen-based applications

▪ What happens if math is misunderstood?

- Human readers
  - confusion
  - frustration
  - headache



- Software tools
  - unexpected results
  - incorrect calculation
  - hidden errors

*Maple 10 - Untitled (1) - [Server 1]

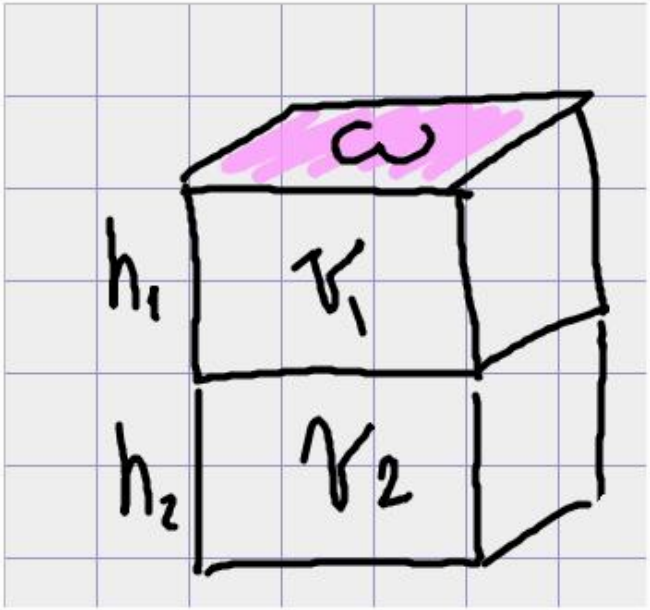
File Edit View Insert Format Table Plot Spreadsheet Sketch Tools Window Help

Text Math Error Monospaced 24

$$h_1 := 5'7"$$

$$h2 := 8'10"$$

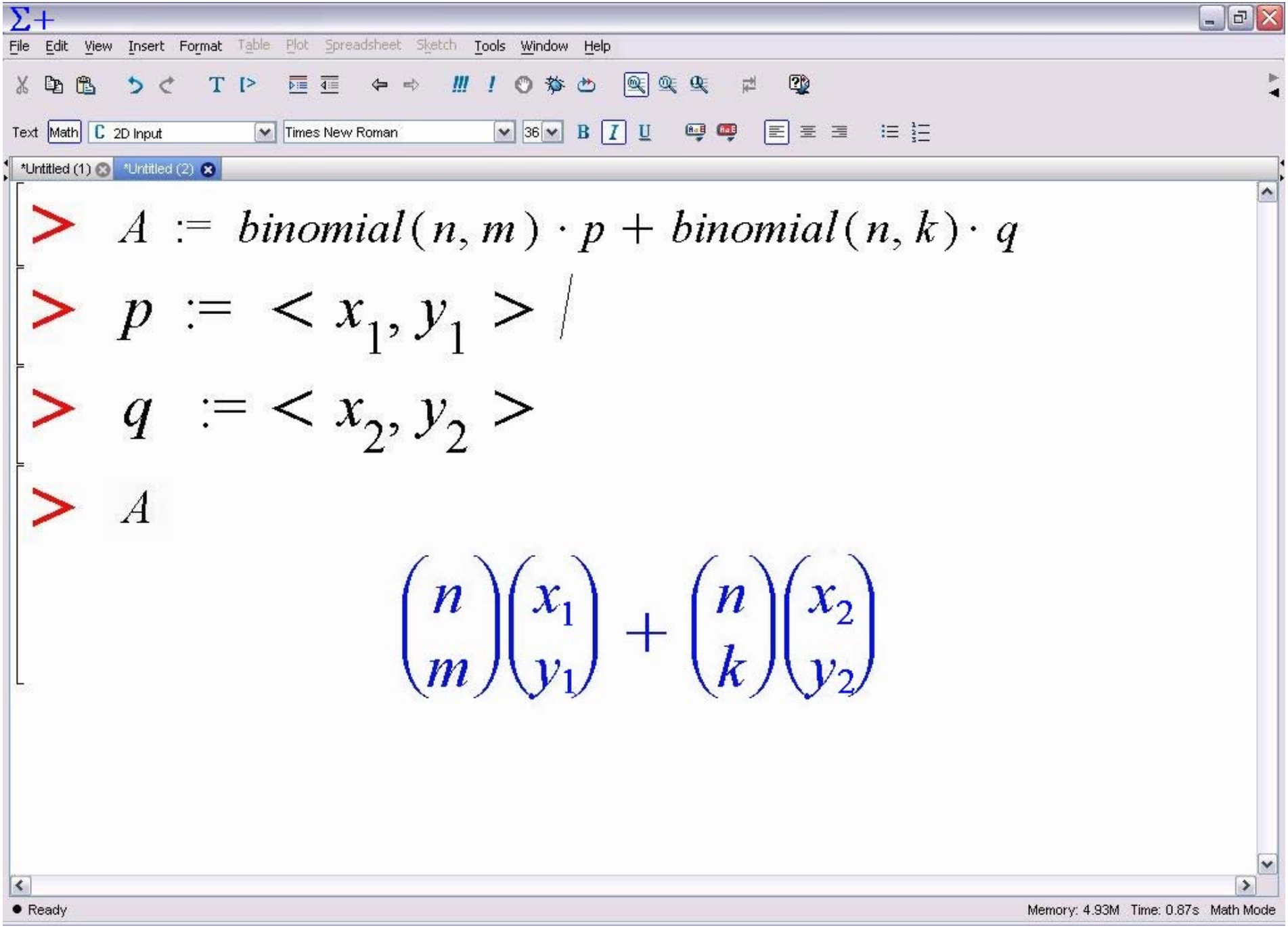
$$S_w = \frac{V_1 + V_2}{(h_1 + h_2)}$$



Error, numeric exception: division by zero

Ready

Memory: 5.62M Time: 9.53s Text Mode


File Edit View Insert Format Table Plot Spreadsheet Sketch Tools Window Help
Text Math 2D Input Times New Roman 36
*Untitled (1) *Untitled (2)
> A := binomial(n, m) · p + binomial(n, k) · q
> p := < x1, y1 >
> q := < x2, y2 >
> A
Ready
Memory: 4.93M Time: 0.87s Math Mode

▪ Mathematical content vs. Notation

Software often confuses

mathematics with ***notation***

▪ Separating notation from content

1. To choose which of several

   different mathematical notations to use

   for *the same concept*.

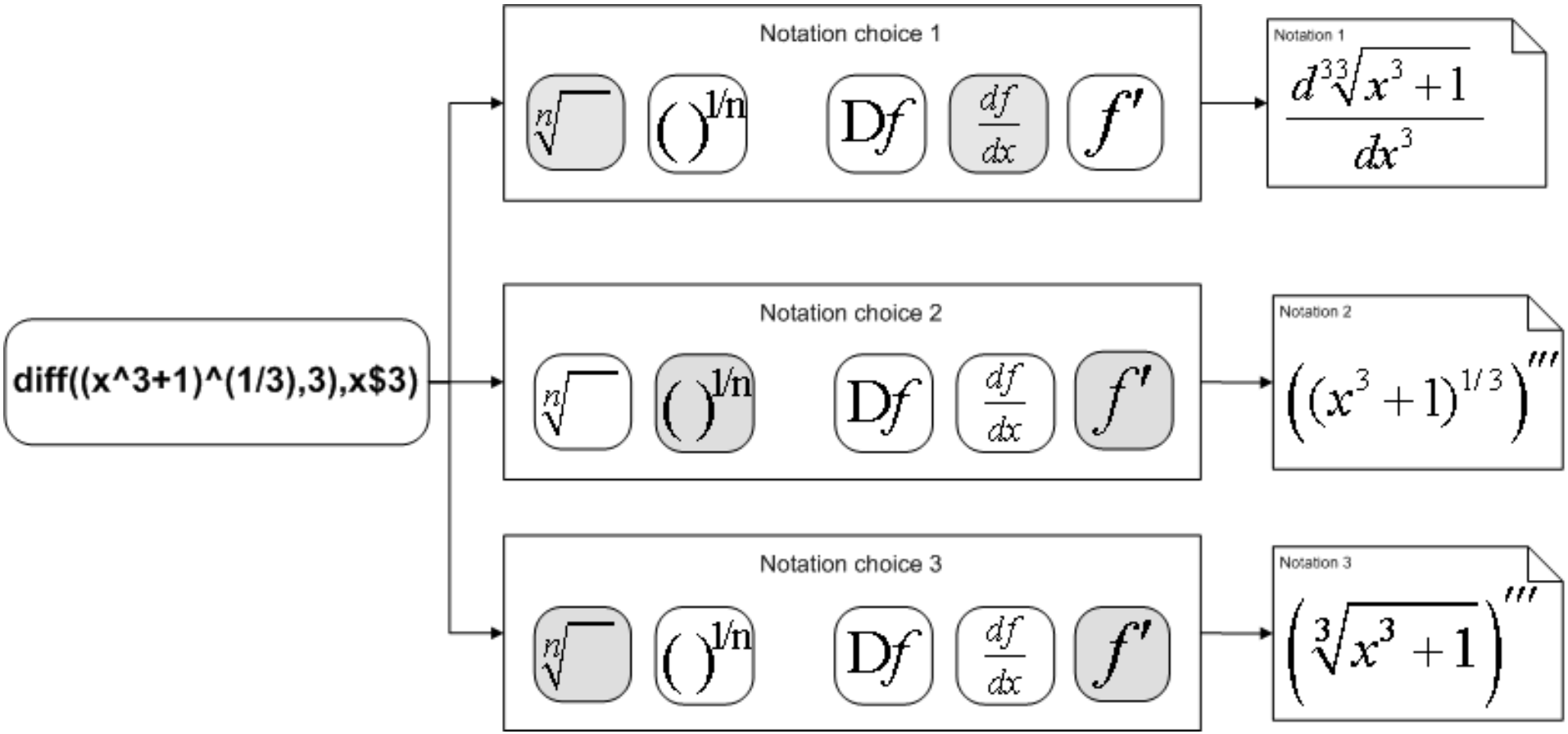
2. To disambiguate where

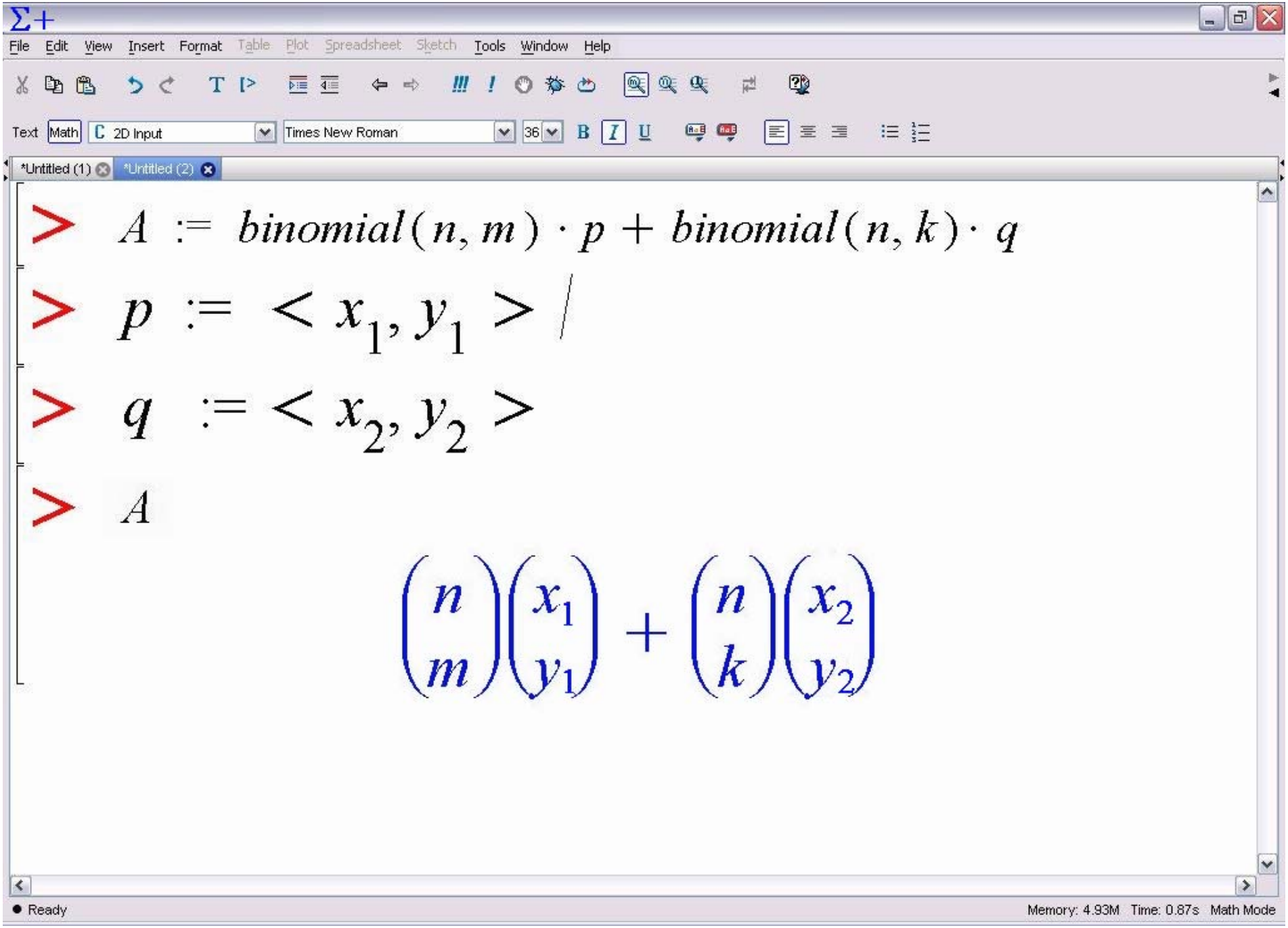
   the *same notation* could be used

   for *different concepts*.

▪ Rendering math with different notations

- Making your math look the way you want

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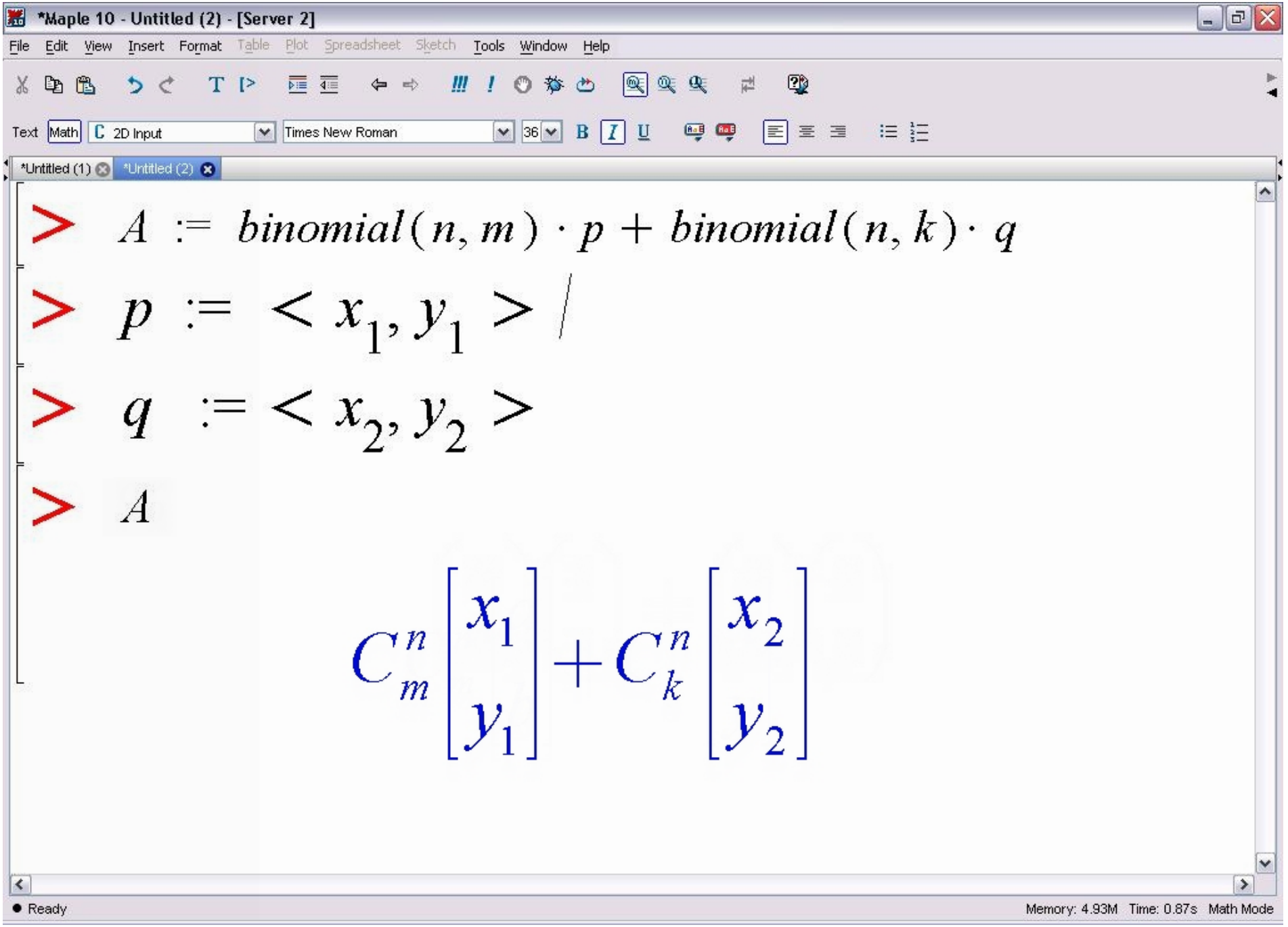
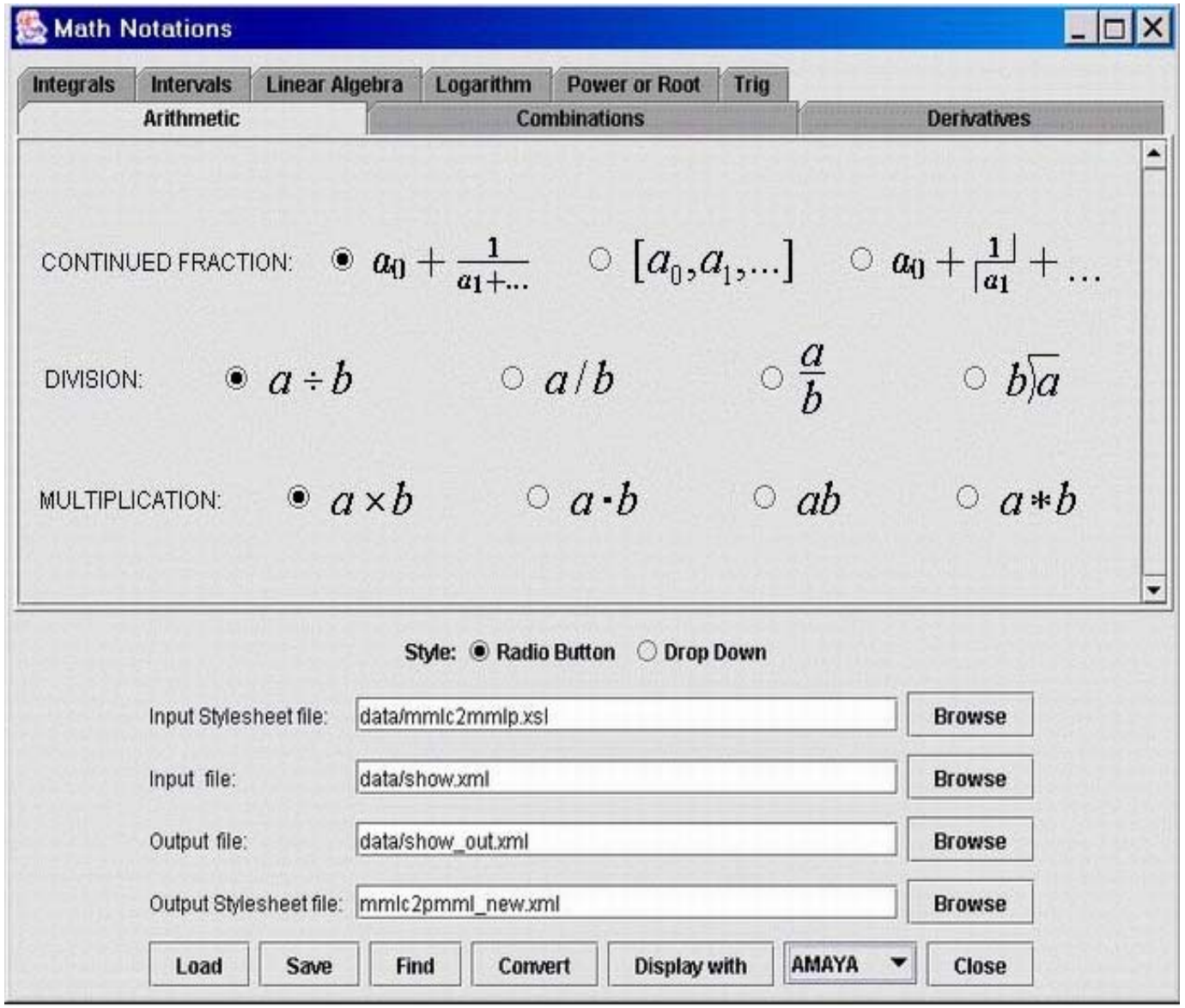
> q := < x2, y2 >
> A
Ready
Memory: 4.93M Time: 0.87s Math Mode


*Maple 10 - Untitled (2) - [Server 2]
File Edit View Insert Format Table Plot Spreadsheet Sketch Tools Window Help
Text Math 2D Input Times New Roman 36
*Untitled (1) *Untitled (2)
> A := binomial(n, m) · p + binomial(n, k) · q
> p := < x1, y1 >
> q := < x2, y2 >
> A
C_m^n [x1, y1] + C_k^n [x2, y2]
Ready
Memory: 4.93M Time: 0.87s Math Mode

▪ A Notation Selection Tool outlook



▪ A Notation Selection Tool spec

- Designed to drive the conversion from *XML-based* ***conceptually*-oriented** *math documents* into ***notationally-oriented*** formats

- It **generates rules** to translate between math expressions in different XML formats

- Main feature: **extensibility –**
  user can introduce additional math concepts along with the set of possible notations

A Notation Selection Tool anatomy

- Java program + web service
- Set of target domains
- Configuration file
- Initial XSLT stylesheet

▪ Notation Selection Tool configuration

- Notation Selection Tool is just a software engine. Its actual behaviour depends on the configuration.

- Configuration defines
  - target mathematical domains,
  - concepts in each domain,
  - set of alternative notations for each concept
  - transformation rules for each notational choice

- Core of the tool is the *configuration file*

The configuration file

```
<catalog>
  <name> LINEAR ALGEBRA </name>
  <itemlist>
    <item>
      <keyword> INNER PRODUCT </keyword>
      <choicelist>
        <choice>
          <image src = "inn_prod1.gif"/>
          <keyvalue> 1 </keyvalue>
          <presentation>
              <converter input="C MathML" output="P MathML>
                ... <!-- XSLT template for this notation-->
            </converter>
          </presentation>
        </choice>
       %other choices,
     %other items,
%other catalogs
```

▪ Features of the Notation Selection Tool

Advantages our this approach:

flexibility and **extensibility**.

- To *introduce new notations* for existing math concepts the user needs simply update Notation Selection Tool configuration file.

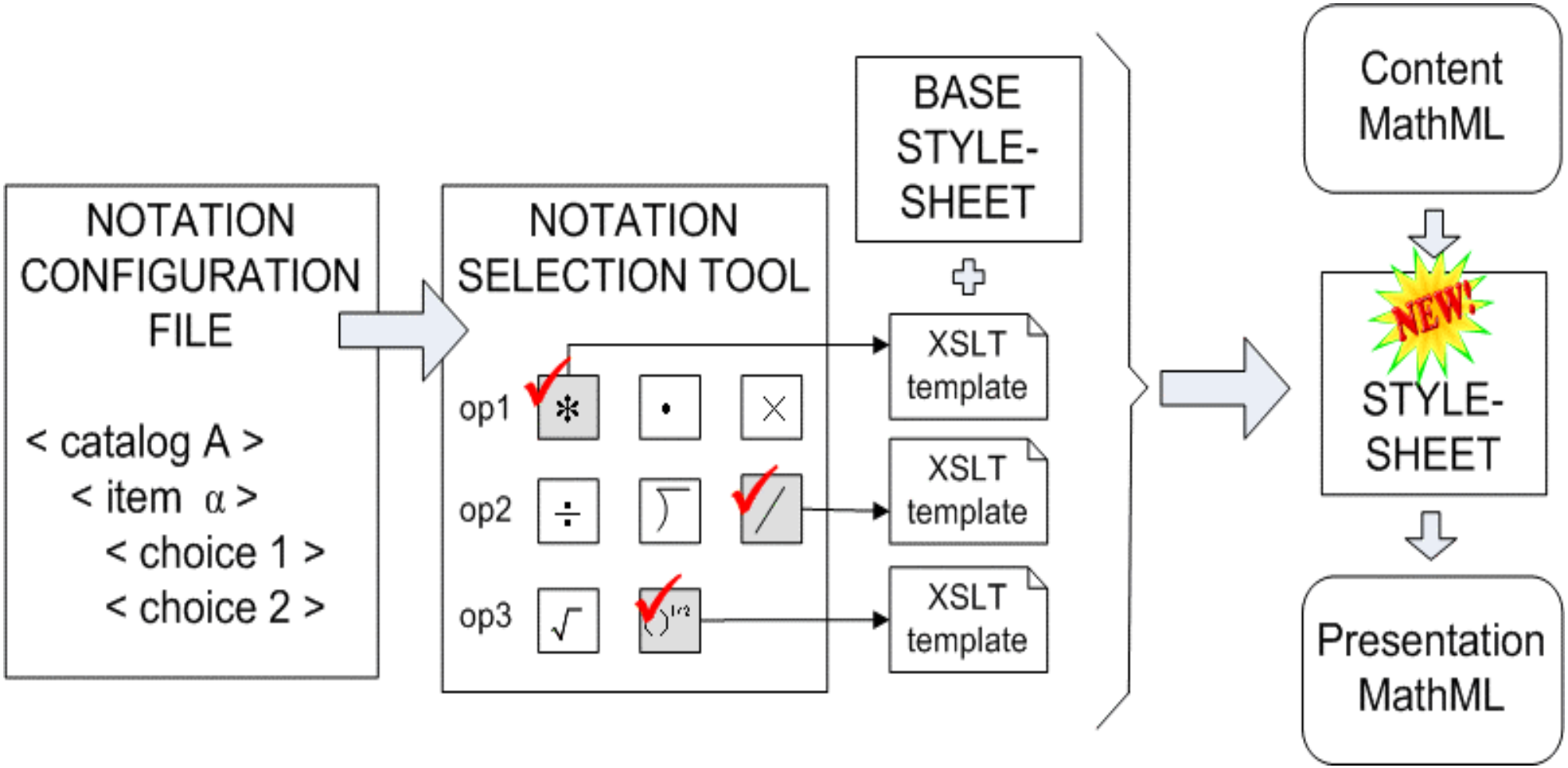
▪ Features of the Notation Selection Tool (2)

Advantages our this approach:

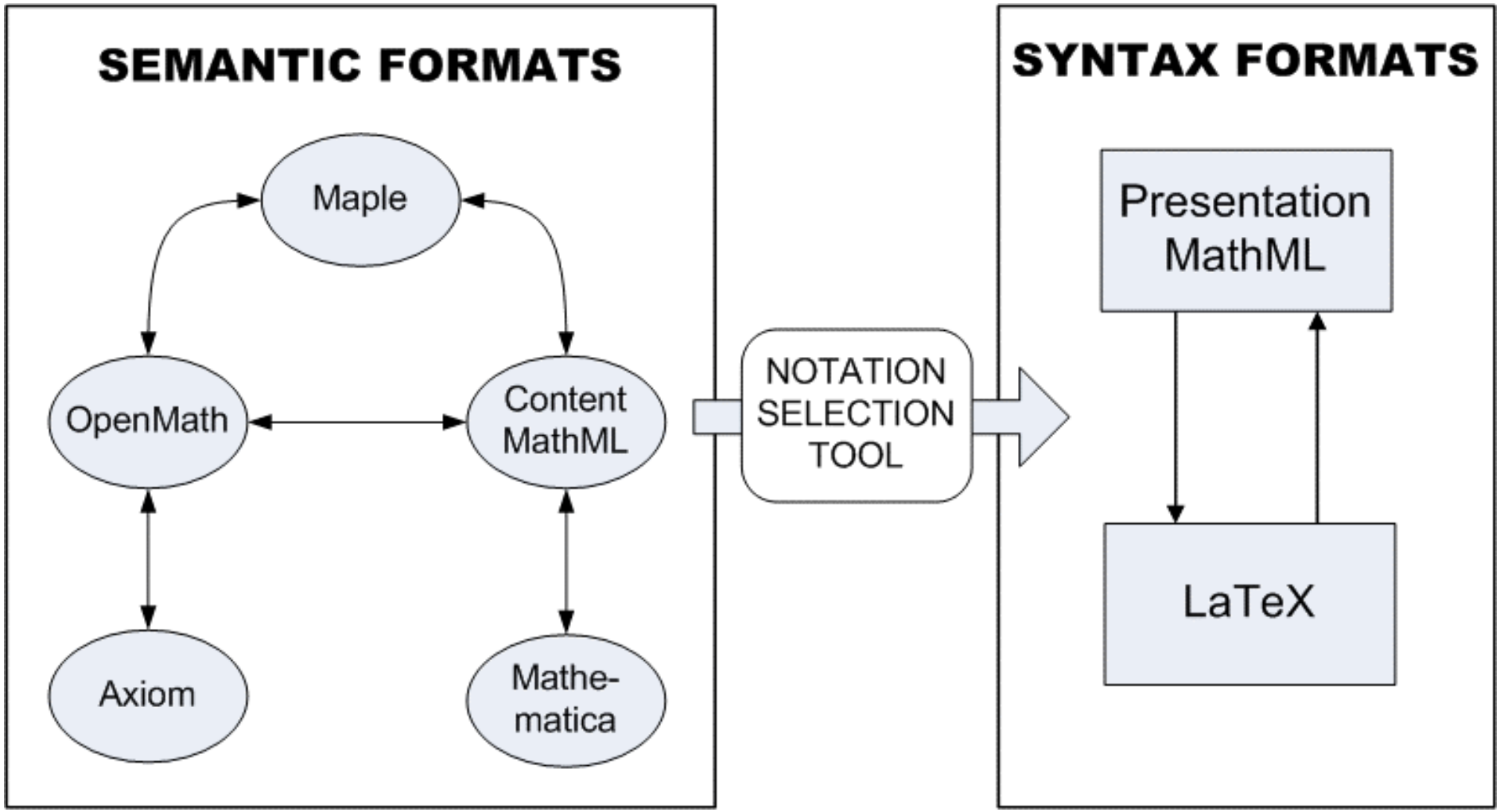
flexibility and **extensibility**.

- *New mathematical concepts* can be introduced in existing settings.
  - e.g. binomial or continued fractions are defined neither in Content MathML, nor in Presentation MathML, but they can be introduced as additional stylesheet templates.
  - The same approach allows to set preferred rendering for OpenMath CDs

▪ Notation Selection Tool in action

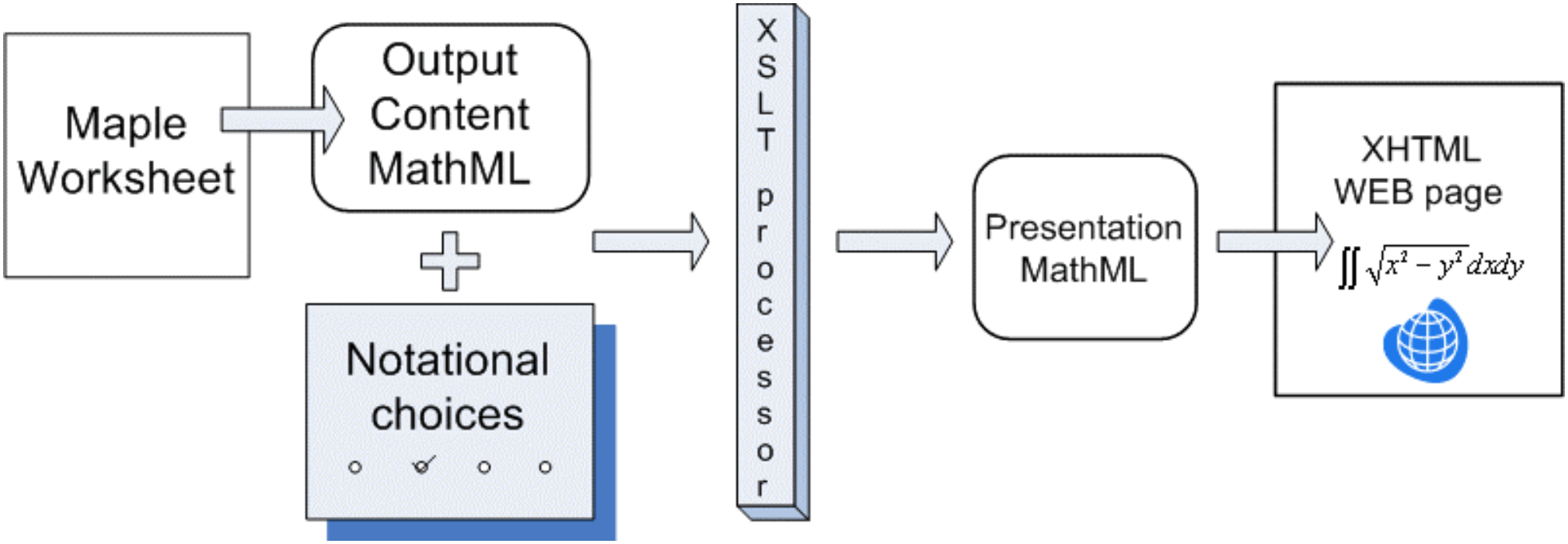


▪ Mathematical Data formats involved



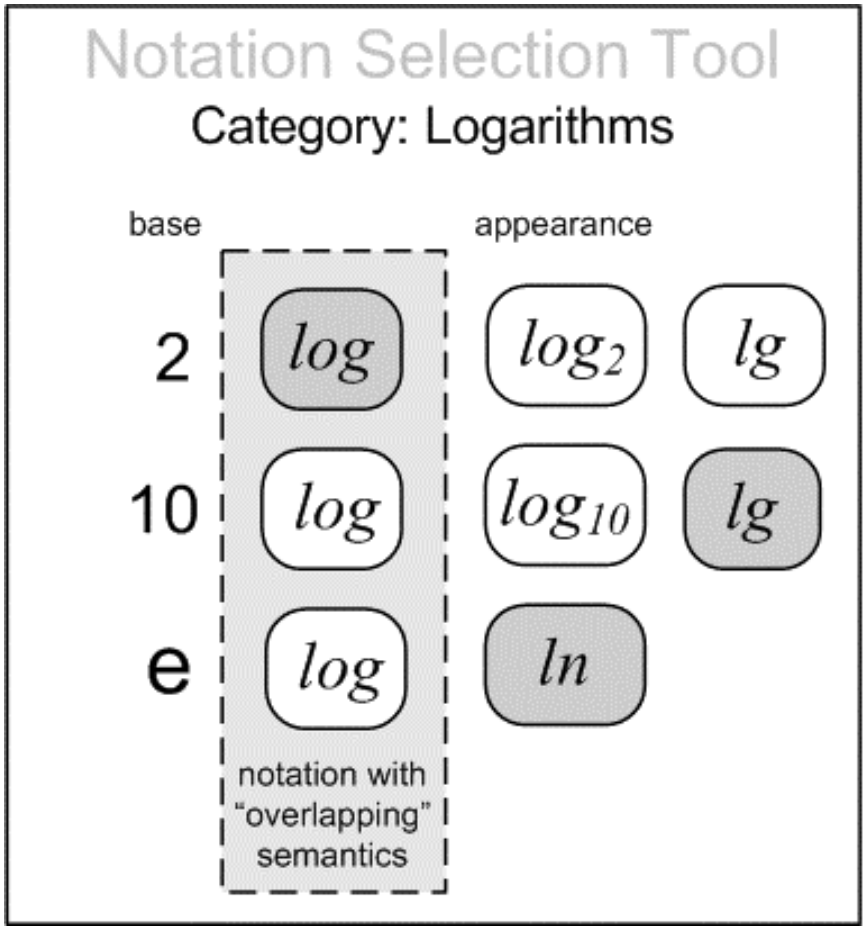
▪ Application of the notation selection (1)

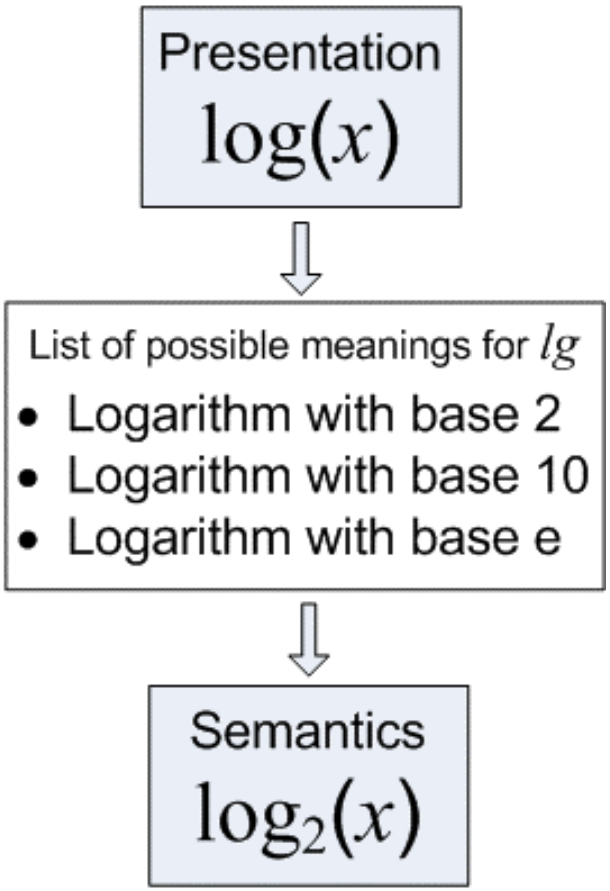
- Rendering mathematical content produced by CAS



▪ Application of the notation selection (2)

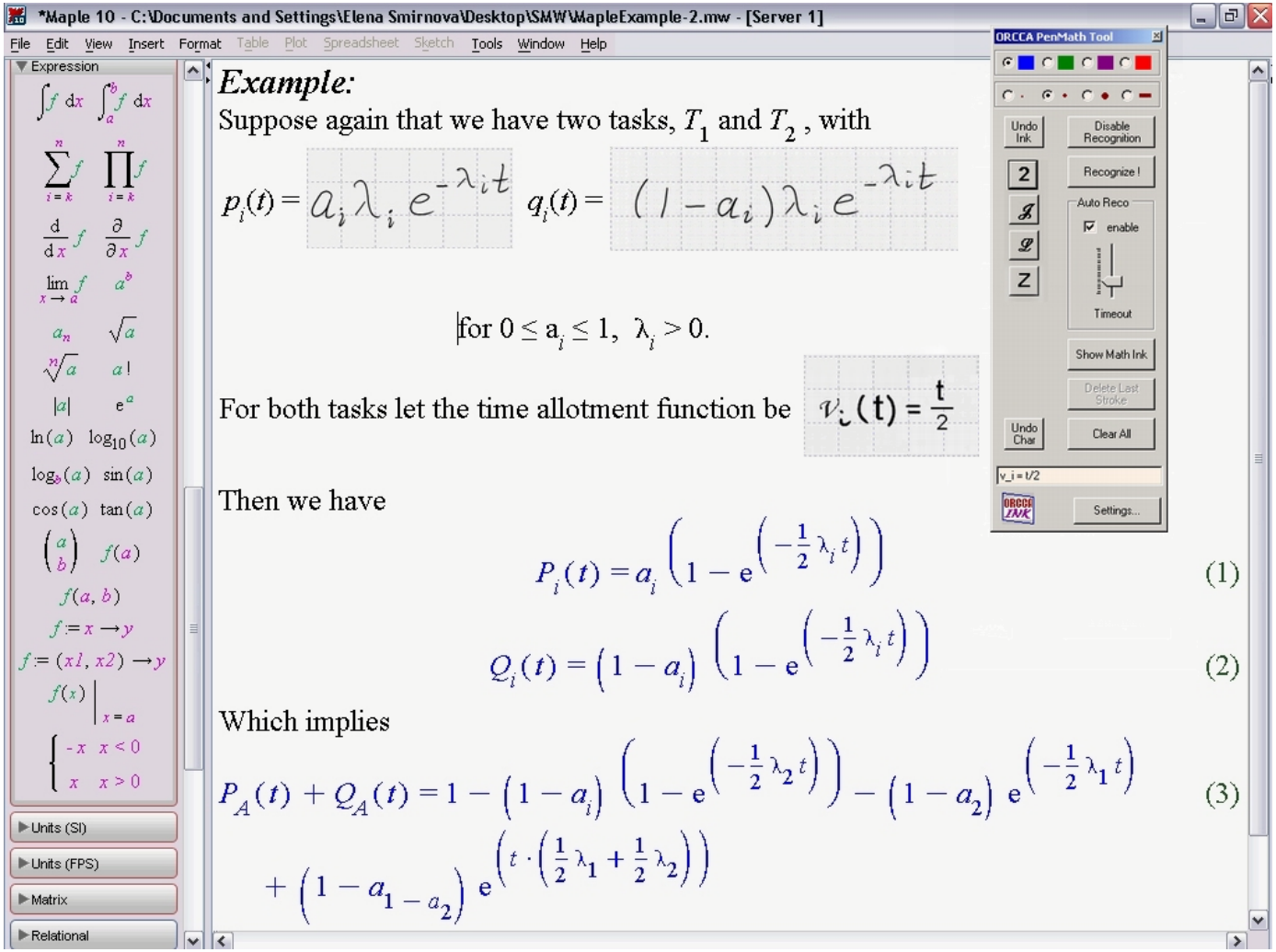
- Syntax disambiguation





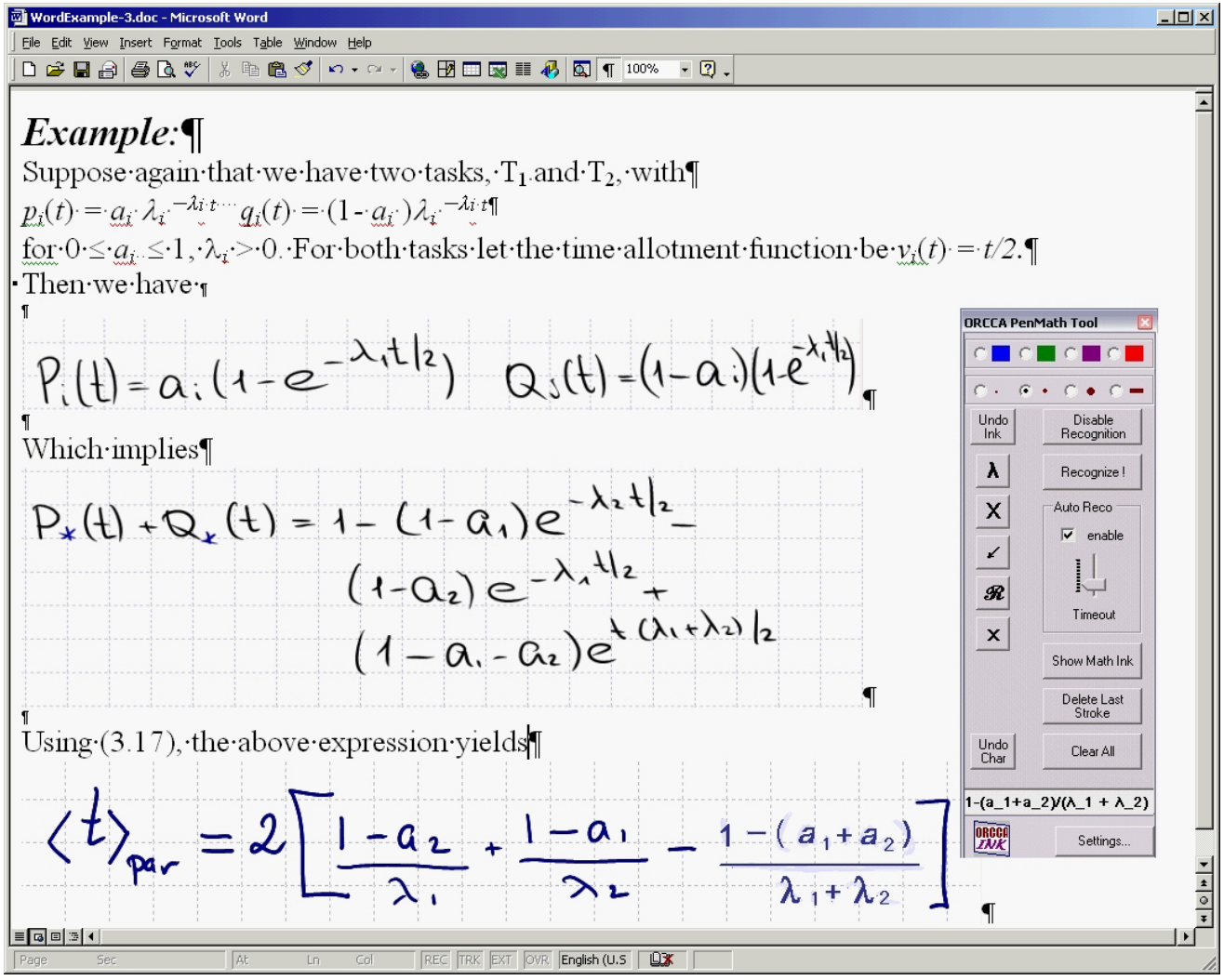
▪ Application of the notation selection (3)

- Mathematical handwriting recognition
  - Computer Algebra Systems



▪ Application of the notation selection (3)

- Mathematical handwriting recognition
  - Document-processing software



▪ Application of the notation selection (3)

- Mathematical handwriting recognition

Helps to narrow down a set of recognition candidates without restricting user's writing habits:

- to eliminate un-used notations.

  Ex. if $\sim$ is chosen for proportionality, then

  $\propto$ will not be suggested as a candidate for α or ∞

- to activate/deactivate various rules in structures recognition.

  Ex.]a,b] or $\int \mathrm{d}x\, f(x)$

▪ Application of the notation selection (4)

- Mathematical knowledge management

  NS used as a front end for mathematical knowledge databases could help to

  - avoid storing duplicated information
  - allow queries without forcing a particular notation

Ex. sinh(x) and *sh(x)*

Should *arcsin(x)* – $sin^{-1}(x)$ simplify to 0?

▪ Application of the notation selection (5)

- Computer Algebra Systems

  The most widely used math software packages are designed with *input* and *output* systems **separate** from one another.

  - NS is aimed to ensure consistency in *both* input and output notations

  - It allows to set the appearance of new math expressions, generated by CAS, such as $J_0(z)$ instead of bulky `BesselJ[0](z)`

▪ Application of the notation selection (sum)

- Rendering mathematical content
- Syntax disambiguation
- Mathematical handwriting recognition
- Mathematical knowledge management
- Computer Algebra Systems

▪ Conclusions

- Notation ambiguity often takes place in mathematics
- Software often confuses mathematics with notation
- By forcing too early the choice of notations, flexibility is lost and work is restricted to too narrow context
- Special tools can be used to translate meaningful math constructs both **to** and **from** a wide range of notations
- This will allow math documents to be deployed in a wide range of settings with notation customized by country, field, level of sophistication etc.

Conclusions

- Notation Selection Tool is an interactive stylesheet constructor dealing with Content and Presentation MathML

- By using it in conjunction with other translators NST covers the following conversions:

Content MathML OpenMath, Maple Axiom Mathematica	→	Presentation MathML LaTeX

- NS is promising to be useful in application to
  - Context of Computer Algebra
  - Syntax disambiguation
  - Pen-based mathematical interfaces
  - Mathematical knowledge management

Related projects at ORCCA

Software for mathematical interfaces

math communication and math format conversions:

- MathML $\leftrightarrow$ LaTeX
- Content MathML $\rightarrow$ Presentation MathML
- OpenMath $\leftrightarrow$ Content MathML
- Maple $\leftrightarrow$ OpenMath
- Pen-Based Interfaces for Mathematics

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