

Νοτατιον Σελεκτιον ιν Μαθηματικαλ Κομπυτινγ Ενπιρονμεντσ

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Notation Selection in Mathematical Computing Environments

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▪ Simple test 1

How do we write the following?

1. the open interval with end points a and b :

2. the base 10 logarithm of x :

3. the inner product of p and q :

4. the arctangent of 2:

▪ Simple test 1

How do we write the following?

1. the open interval with end points a and b : $]a, b[$

2. the base 10 logarithm of x : $\lg x$

3. the inner product of p and q : $p \perp q$

4. the arctangent of 2: $\operatorname{tg}^{-1} 2$

▪ Simple test 2

What does it mean for you?

1. $\lg x$

2. $\tan^{-1} a$

3. $\left(\frac{p}{q}\right)$

4. u'

▪ Simple test 2

What does it mean for you?

1. $\lg x$
- $\log_{10} x$
 - $\log_2 x$

2. $\tan^{-1} a$
- cotangent a
 - arctangent a

3. $\left(\frac{p}{q}\right)$
- Legendre symbol
 - Parenthesized fraction

4. u'
- derivative of u
 - u minutes
 - transformation performed on an original u

▪ Origin of notational ambiguity

- **Mathematical context**

- an ordinary derivative can be denoted as

$$f', f_x, Df, \frac{df}{dx} \text{ etc.}$$

- **Area of application**

- i for $\sqrt{-1}$ (complex analysis) vs.
 j (electrical engineering).

- the integral

$$\int f(x)dx \text{ (pure math) vs. } \int dx f(x) \text{ (physics)}$$

▪ Origin of notational ambiguity (cont-d)

- **National and cultural conventions**

- the tangent function:

\tan (England, Canada) vs. tg (Russia, China);

- the open interval:

(a, b) (United States) vs. $]a, b[$ (France).

- **Historical period**

- e.g. ancient $\sqrt[3]{a+b}$ vs. the modern $\sqrt[3]{a+b}$.

- **Level of mathematical sophistication**

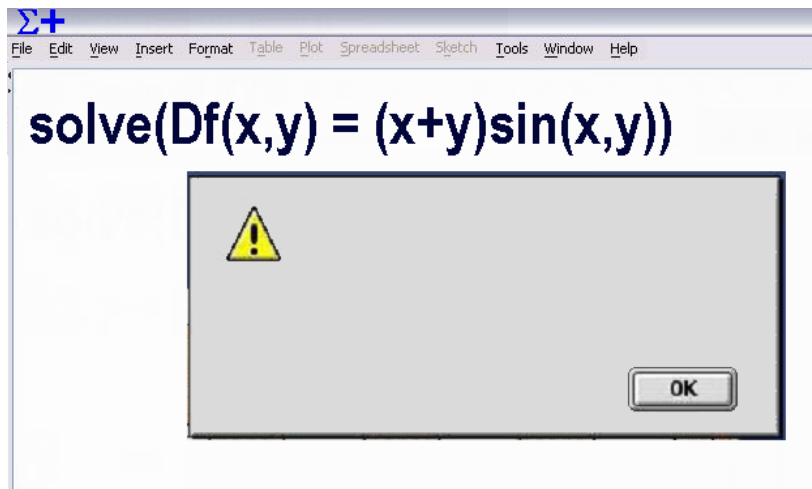
- e.g. $a \div b$ vs. $b \overline{)a}$ vs. $\frac{a}{b}$ vs. a/b vs. $\frac{a}{/b}$.

▪ Who needs to understand written math?

- Human readers
- Software tools:
 - CAS,
 - theorem provers,
 - format converters,
 - pen-based applications

▪ What happens if math is misunderstood?

- Human readers
 - confusion
 - frustration
 - headache

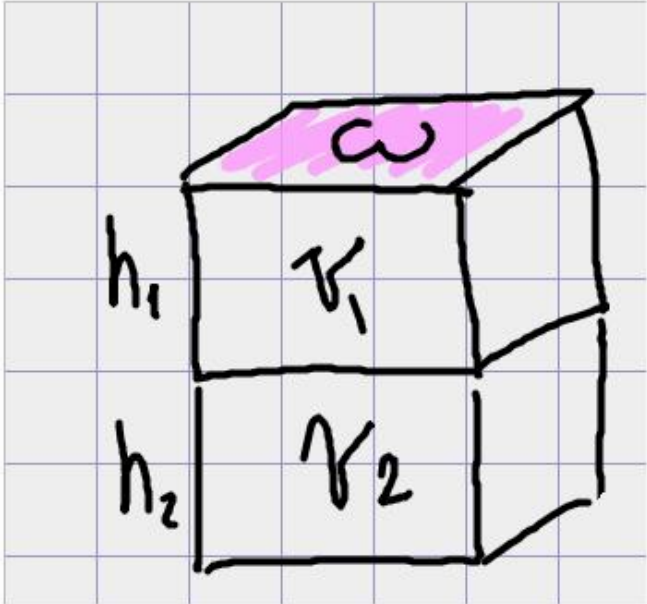


- Software tools
 - unexpected results
 - incorrect calculation
 - hidden errors

*Maple 10 - Untitled (1) - [Server 1]

File Edit View Insert Format Table Plot Spreadsheet Sketch Tools Window Help

Text Math P Error Monospaced 24 B I U

$$h_1 := 5'7''$$
$$h_2 := 8'10''$$
$$S_w = \frac{V_1 + V_2}{\left(\begin{matrix} h_1 & + & h_2 \\ 1 & & 2 \end{matrix} \right)}$$


Error, numeric exception:
division by zero

Ready Memory: 5.62M Time: 9.53s Text Mode

$\Sigma+$

File Edit View Insert Format Table Plot Spreadsheet Sketch Tools Window Help

Text Math 2D Input Times New Roman 36 B I U

*Untitled (1) *Untitled (2)

$\succ A := \text{binomial}(n, m) \cdot p + \text{binomial}(n, k) \cdot q$

$\succ p := \langle x_1, y_1 \rangle$

$\succ q := \langle x_2, y_2 \rangle$

$\succ A$

$$\binom{n}{m} \binom{x_1}{y_1} + \binom{n}{k} \binom{x_2}{y_2}$$

Ready Memory: 4.93M Time: 0.87s Math Mode

▪ Mathematical content vs. Notation



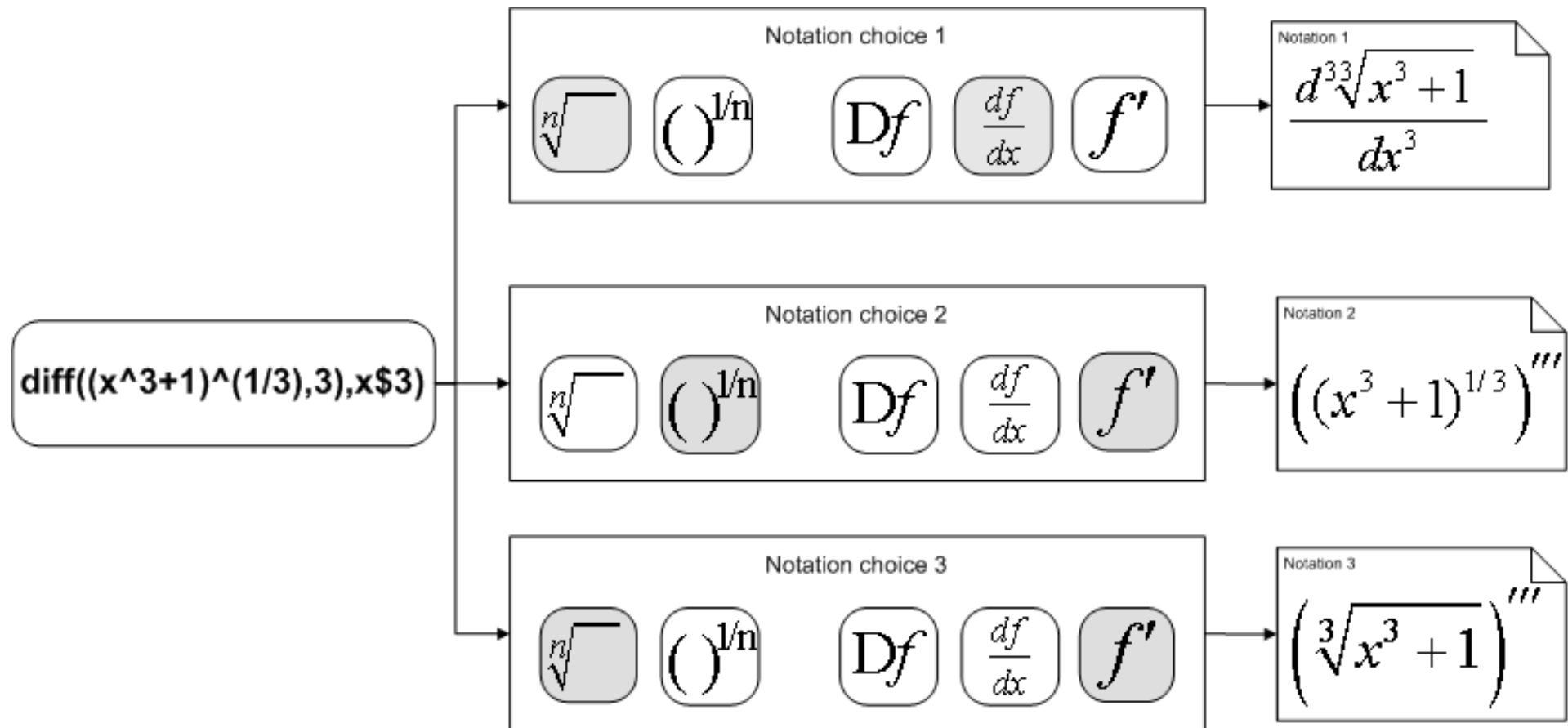
Software often confuses
mathematics with *notation*

▪ Separating notation from content

1. To choose which of several *different mathematical notations* to use for *the same concept*.
2. To disambiguate where the *same notation* could be used for *different concepts*.

Rendering math with different notations

- Making your math look the way you want



Σ+

File Edit View Insert Format Table Plot Spreadsheet Sketch Tools Window Help

Text Math 2D Input Times New Roman 36 B I U

*Untitled (1) *Untitled (2)

> $A := \text{binomial}(n, m) \cdot p + \text{binomial}(n, k) \cdot q$

> $p := \langle x_1, y_1 \rangle$ /

> $q := \langle x_2, y_2 \rangle$

> A

$$\binom{n}{m} \binom{x_1}{y_1} + \binom{n}{k} \binom{x_2}{y_2}$$

Ready Memory: 4.93M Time: 0.87s Math Mode

*Maple 10 - Untitled (2) - [Server 2]

File Edit View Insert Format Table Plot Spreadsheet Sketch Tools Window Help

Text Math 2D Input Times New Roman 36 B I U

*Untitled (1) *Untitled (2)

> $A := \text{binomial}(n, m) \cdot p + \text{binomial}(n, k) \cdot q$

> $p := \langle x_1, y_1 \rangle /$

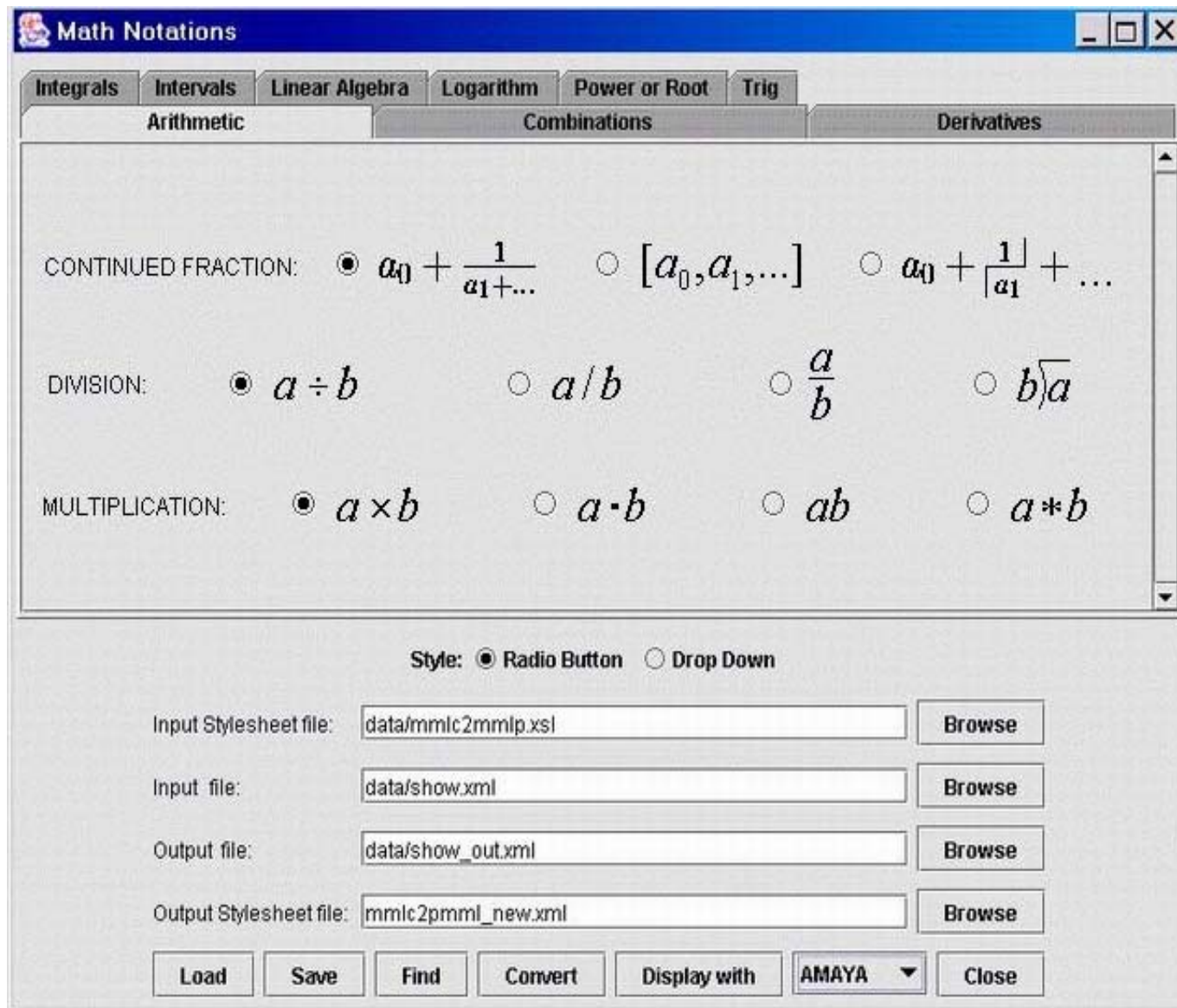
> $q := \langle x_2, y_2 \rangle$

> A

$$C_m^n \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + C_k^n \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

Ready Memory: 4.93M Time: 0.87s Math Mode

▪ A Notation Selection Tool outlook



▪ A Notation Selection Tool spec

- Designed to drive the conversion from *XML-based conceptually-oriented math documents* into *notationally-oriented* formats
- It **generates rules** to translate between math expressions in different XML formats
- Main feature: **extensibility** – user can introduce additional math concepts along with the set of possible notations

▪ A Notation Selection Tool anatomy

- Java program + web service
- Set of target domains
- Configuration file
- Initial XSLT stylesheet



▪ Notation Selection Tool configuration

- Notation Selection Tool is just a software engine. Its actual behaviour depends on the configuration.
- Configuration defines
 - target mathematical domains,
 - concepts in each domain,
 - set of alternative notations for each concept
 - transformation rules for each notational choice
- Core of the tool is the *configuration file*

▪ The configuration file

```
<catalog>
  <name> LINEAR ALGEBRA </name>
  <itemlist>
    <item>
      <keyword> INNER PRODUCT </keyword>
      <choicelist>
        <choice>
          <image src = "inn_prod1.gif"/>
          <keyvalue> 1 </keyvalue>
          <presentation>
            <converter input="C MathML" output="P MathML">
              ... <!-- XSLT template for this notation-->
            </converter>
          </presentation>
        </choice>
        %other choices,
      %other items,
    %other catalogs
```

▪ Features of the Notation Selection Tool

Advantages of this approach:

flexibility and extensibility.

- To *introduce new notations* for existing math concepts the user needs simply update Notation Selection Tool configuration file.

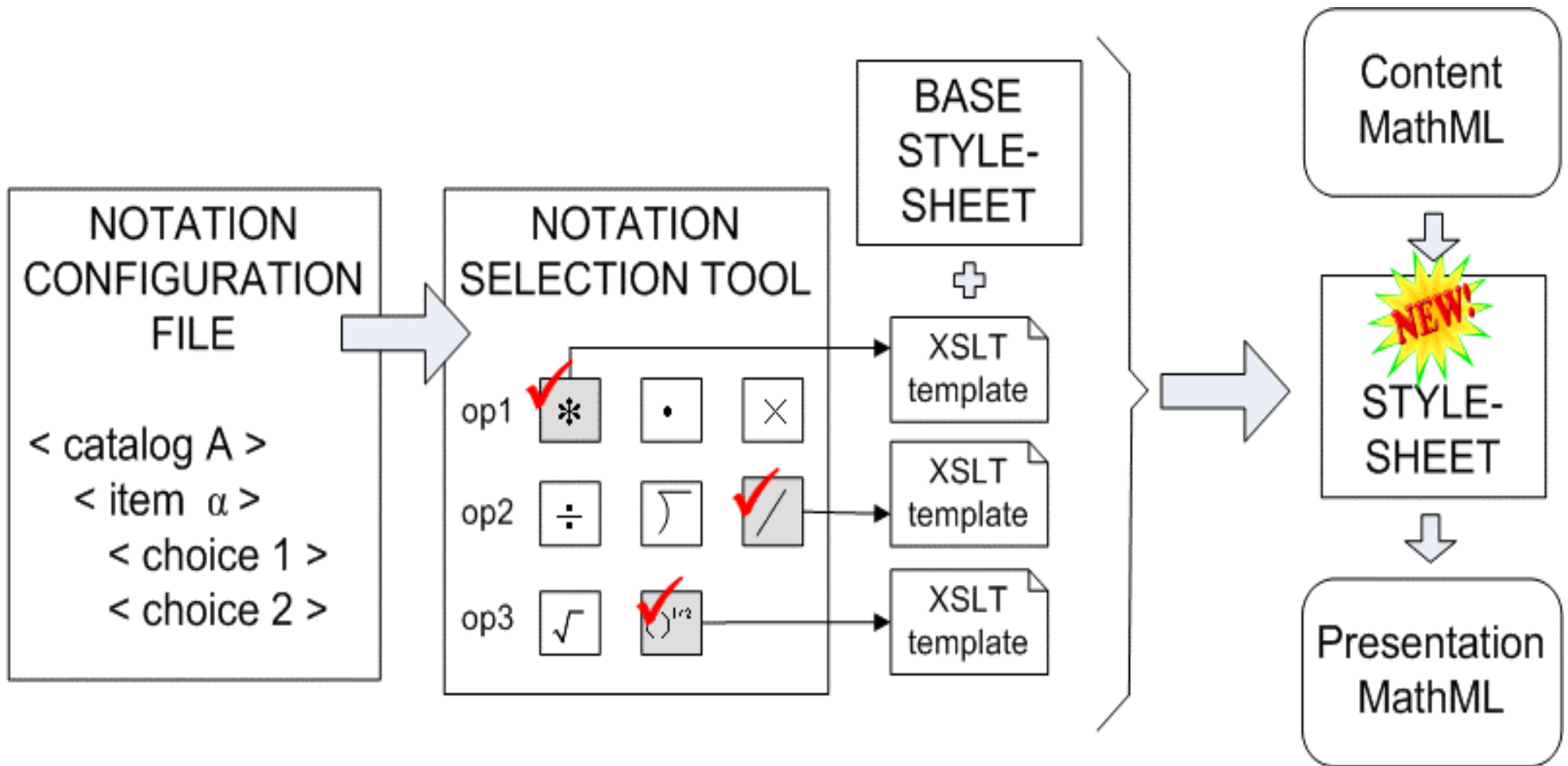
▪ Features of the Notation Selection Tool (2)

Advantages of this approach:

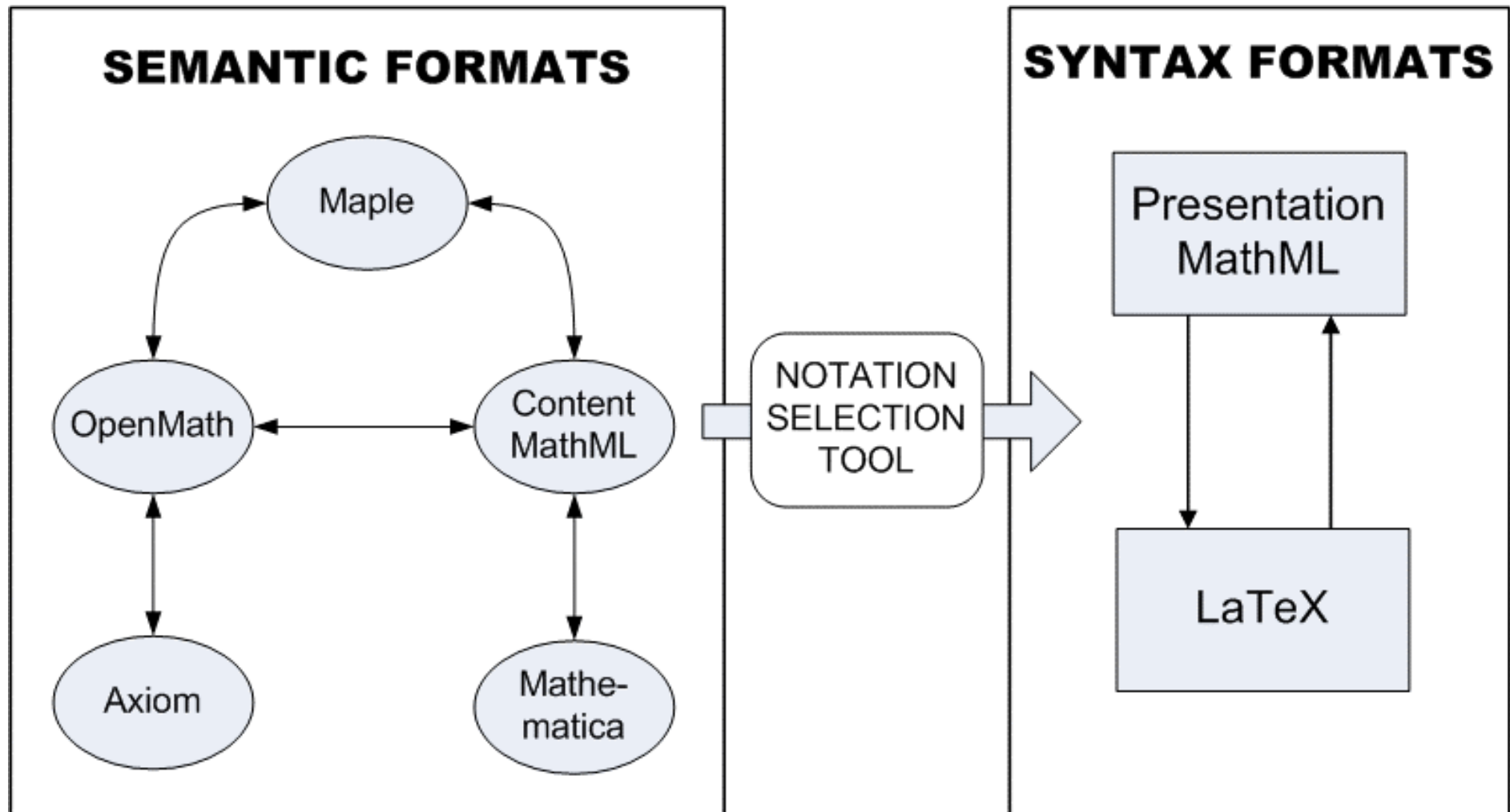
flexibility and extensibility.

- *New mathematical concepts* can be introduced in existing settings.
 - e.g. binomial or continued fractions are defined neither in Content MathML, nor in Presentation MathML, but they can be introduced as additional stylesheet templates.
 - The same approach allows to set preferred rendering for OpenMath CDs

Notation Selection Tool in action

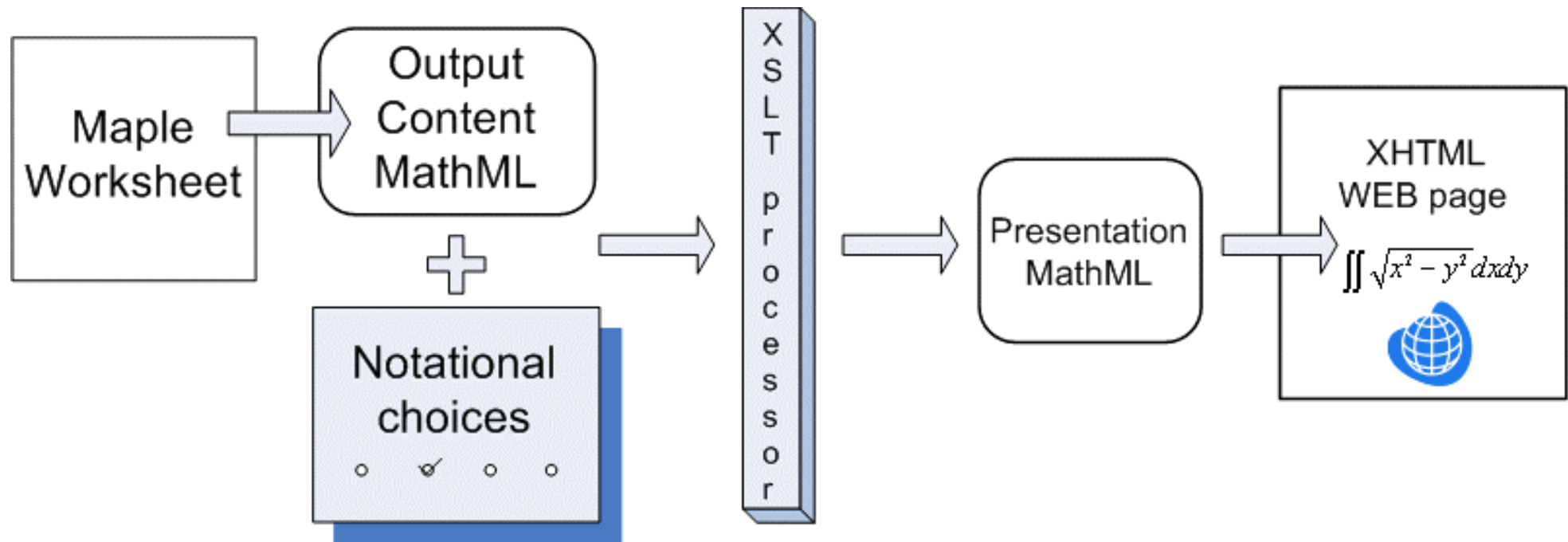


Mathematical Data formats involved



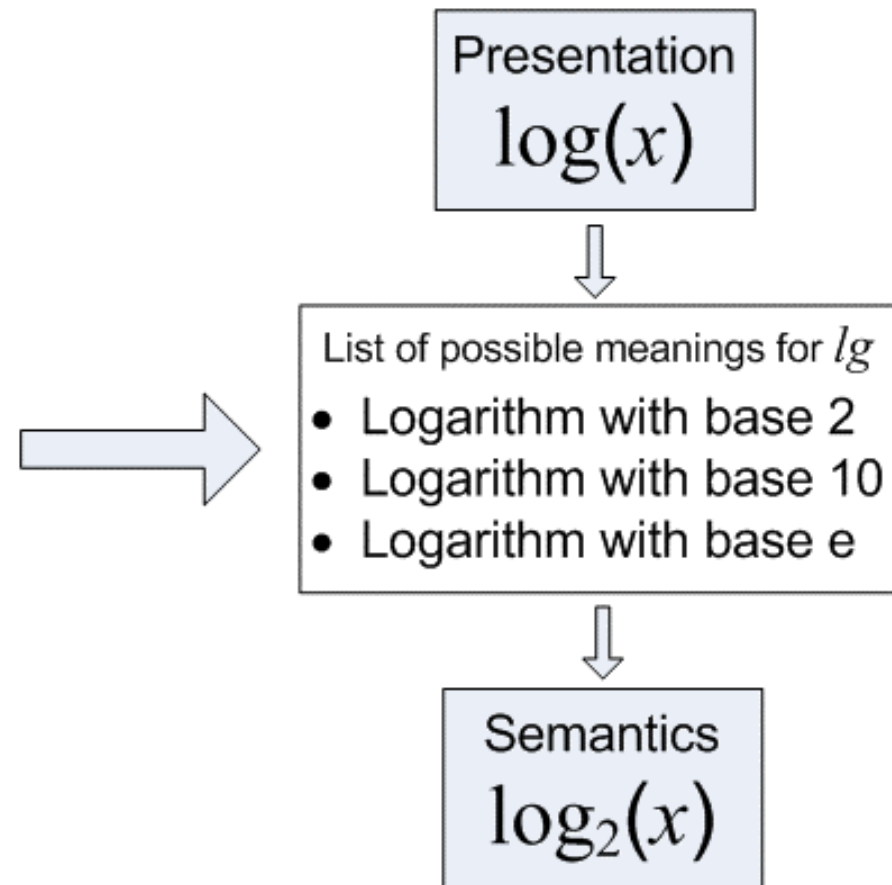
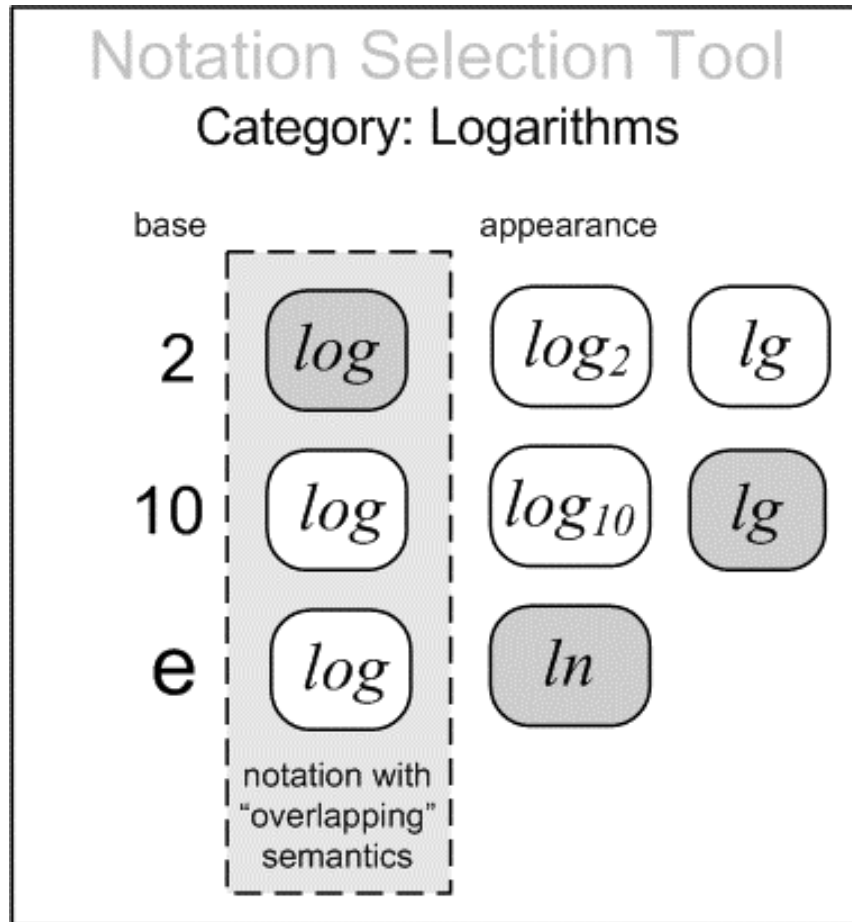
Application of the notation selection (1)

- Rendering mathematical content produced by CAS



Application of the notation selection (2)

- Syntax disambiguation



Application of the notation selection (3)

- Mathematical handwriting recognition
 - Computer Algebra Systems

Example:
 Suppose again that we have two tasks, T_1 and T_2 , with

$$p_i(t) = a_i \lambda_i e^{-\lambda_i t} \quad q_i(t) = (1 - a_i) \lambda_i e^{-\lambda_i t}$$

for $0 \leq a_i \leq 1, \lambda_i > 0$.

For both tasks let the time allotment function be $v_i(t) = \frac{t}{2}$

Then we have

$$P_i(t) = a_i \left(1 - e^{-\frac{1}{2} \lambda_i t} \right) \tag{1}$$

$$Q_i(t) = (1 - a_i) \left(1 - e^{-\frac{1}{2} \lambda_i t} \right) \tag{2}$$

Which implies

$$P_A(t) + Q_A(t) = 1 - (1 - a_1) \left(1 - e^{-\frac{1}{2} \lambda_2 t} \right) - (1 - a_2) e^{-\frac{1}{2} \lambda_1 t} \tag{3}$$

$$+ (1 - a_1 - a_2) e^{t \cdot \left(\frac{1}{2} \lambda_1 + \frac{1}{2} \lambda_2 \right)}$$

The screenshot also shows a control panel for the ORCCA PenMath Tool with buttons for 'Undo Ink', 'Recognize!', 'Auto Reco', 'Show Math Ink', 'Delete Last Stroke', 'Clear All', and 'Settings...'. The 'Auto Reco' checkbox is checked.

Application of the notation selection (3)

- Mathematical handwriting recognition
 - Document-processing software

The screenshot shows a Microsoft Word document titled "WordExample-3.doc" with handwritten mathematical text. The text includes an example problem and its solution. The ORCCA PenMath Tool interface is visible on the right side of the document, showing various controls for handwriting recognition and editing.

Example:
 Suppose again that we have two tasks, T_1 and T_2 , with
 $p_i(t) = a_i \lambda_i^{-\lambda_i t}$ and $q_i(t) = (1 - a_i) \lambda_i^{-\lambda_i t}$
 for $0 \leq a_i \leq 1$, $\lambda_i > 0$. For both tasks let the time allotment function be $v_i(t) = t/2$.
 Then we have

$$P_i(t) = a_i (1 - e^{-\lambda_i t/2}) \quad Q_i(t) = (1 - a_i) (1 - e^{-\lambda_i t/2})$$

Which implies

$$P_*(t) + Q_*(t) = 1 - (1 - a_1) e^{-\lambda_2 t/2} - (1 - a_2) e^{-\lambda_1 t/2} + (1 - a_1 - a_2) e^{-(\lambda_1 + \lambda_2) t/2}$$

Using (3.17), the above expression yields

$$\langle t \rangle_{par} = 2 \left[\frac{1 - a_2}{\lambda_1} + \frac{1 - a_1}{\lambda_2} - \frac{1 - (a_1 + a_2)}{\lambda_1 + \lambda_2} \right]$$

The ORCCA PenMath Tool interface includes the following elements:

- Color selection buttons (blue, green, purple, red)
- Undo Ink button
- Disable Recognition button
- Recognize! button
- Auto Reco section with an "enable" checkbox and a Timeout slider
- Show Math Ink button
- Delete Last Stroke button
- Clear All button
- Undo Char button
- Settings... button

▪ Application of the notation selection (3)

- Mathematical handwriting recognition

Helps to narrow down a set of recognition candidates without restricting user's writing habits:

- to eliminate un-used notations.

Ex. if \sim is chosen for proportionality, then

\propto will not be suggested as a candidate for α **Or** ∞

- to activate/deactivate various rules in structures recognition.

Ex. $]a,b]$ or $\int dx f(x)$

▪ Application of the notation selection (4)

- Mathematical knowledge management

NS used as a front end for mathematical knowledge databases could help to

- avoid storing duplicated information
- allow queries without forcing a particular notation

Ex. $\sinh(x)$ and $sh(x)$

Should $\arcsin(x) - \sin^{-1}(x)$ simplify to 0?

▪ Application of the notation selection (5)

- Computer Algebra Systems

The most widely used math software packages are designed with *input* and *output* systems **separate** from one another.

- NS is aimed to ensure consistency in *both* input and output notations
- It allows to set the appearance of new math expressions, generated by CAS, such as $J_0(z)$ instead of bulky `BesselJ[0](z)`

▪ Application of the notation selection (sum)

- Rendering mathematical content
- Syntax disambiguation
- Mathematical handwriting recognition
- Mathematical knowledge management
- Computer Algebra Systems

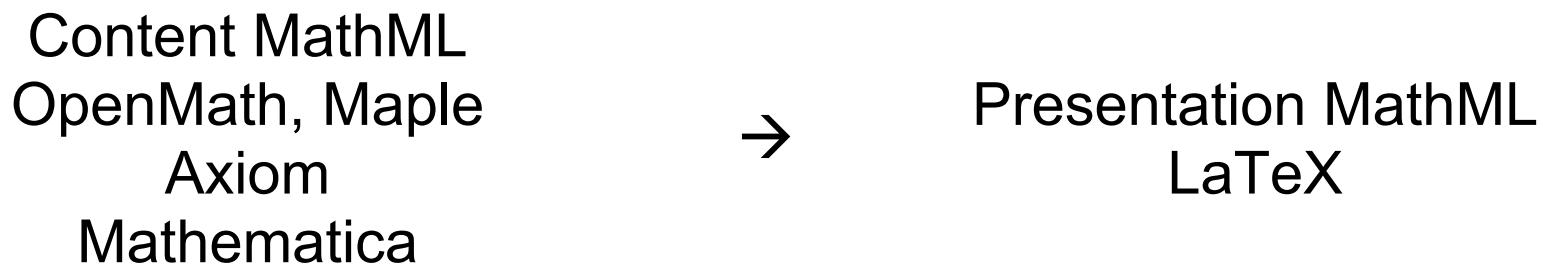
▪ Conclusions

- Notation ambiguity often takes place in mathematics
- Software often confuses mathematics with notation
- By forcing too early the choice of notations, flexibility is lost and work is restricted to too narrow context
- Special tools can be used to translate meaningful math constructs both **to** and **from** a wide range of notations
- This will allow math documents to be deployed in a wide range of settings with notation customized by country, field, level of sophistication etc.

▪ Conclusions

- Notation Selection Tool is an interactive stylesheet constructor dealing with Content and Presentation MathML

- By using it in conjunction with other translators NST covers the following conversions:



- NS is promising to be useful in application to
 - Context of Computer Algebra
 - Syntax disambiguation
 - Pen-based mathematical interfaces
 - Mathematical knowledge management

▪ Related projects at ORCCA

Software for mathematical interfaces

math communication and math format conversions:

- MathML \leftrightarrow LaTeX
- Content MathML \rightarrow Presentation MathML
- OpenMath \leftrightarrow Content MathML
- Maple \leftrightarrow OpenMath
- Pen-Based Interfaces for Mathematics

<http://www.orcca.on.ca>