

MathBrush

A Pen-Based Math System

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Project Team

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How Would You Like to Do Math?

$$\int \frac{(3x^2 + 2) \sin(x^3 + 2x - 1)}{\cos(x^3 + 2x - 1)^3 + 12 \cos(x^3 + 2x - 1)^2 - 3} dx$$

- Latex

```
\begin{equation}
\int \frac{\left( 3x^2 + 2 \right) \sin \left( x^3 + 2x - 1 \right)}{\cos \left( x^3 + 2x - 1 \right)^3 + 12 \cos \left( x^3 + 2x - 1 \right)^2 - 3} dx
\end{equation}
```

- Maple

```
int((3*x^2+2)*sin(x^3+2*x-1)/(cos(x^3+2*x-1)^3 + 12*cos(x^3+2*x-1)^2-3),x);
```

- Mathematica

```
Integrate[(3*x^2+2)*Sin[x^3+2*x-1]/(Cos[x^3+2*x-1]^3 + 12*Cos[x^3+2*x-1]^2-3),x]
```

I wish I can do it the way I do it on a paper, get some math help, and play with it !!

Objective

- Investigate the use of pen-based devices for mathematical computation and exploration
- Study the key issues when combining **pen-based interfaces** with **Computer Algebra Systems (CAS)**
- Build a pen-based math system to allow the user to follow a pen and paper paradigm for input and output while taking advantage of the power of CAS

Motivation

- Entering mathematical concepts on a computer for presentation is tedious and difficult. Ex. using MathML and Latex
- The current interface of CAS, although has improved immensely, it constraints the user to express his thoughts in a command line form
- Different CAS systems have different languages and commands
- Manipulation and transformation of expressions in CAS lack intuitiveness when using only keyboard and mouse

Challenges

- Text recognizers are not suitable:
 - work with ASCII Characters
 - depend on language specific dictionaries
 - assume input is one dimension
- Mathematical symbols are more ambiguous than regular text

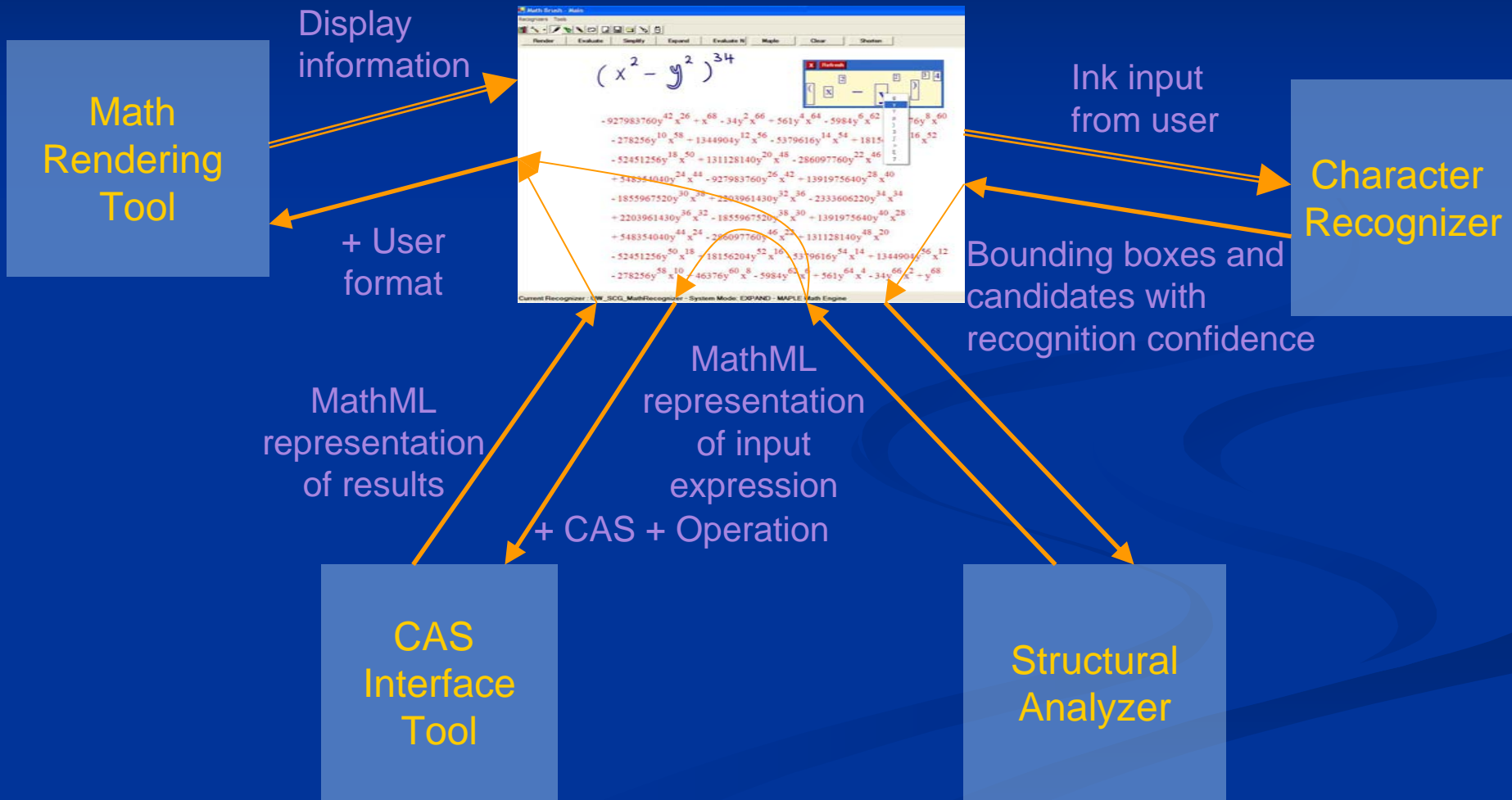
Ex.



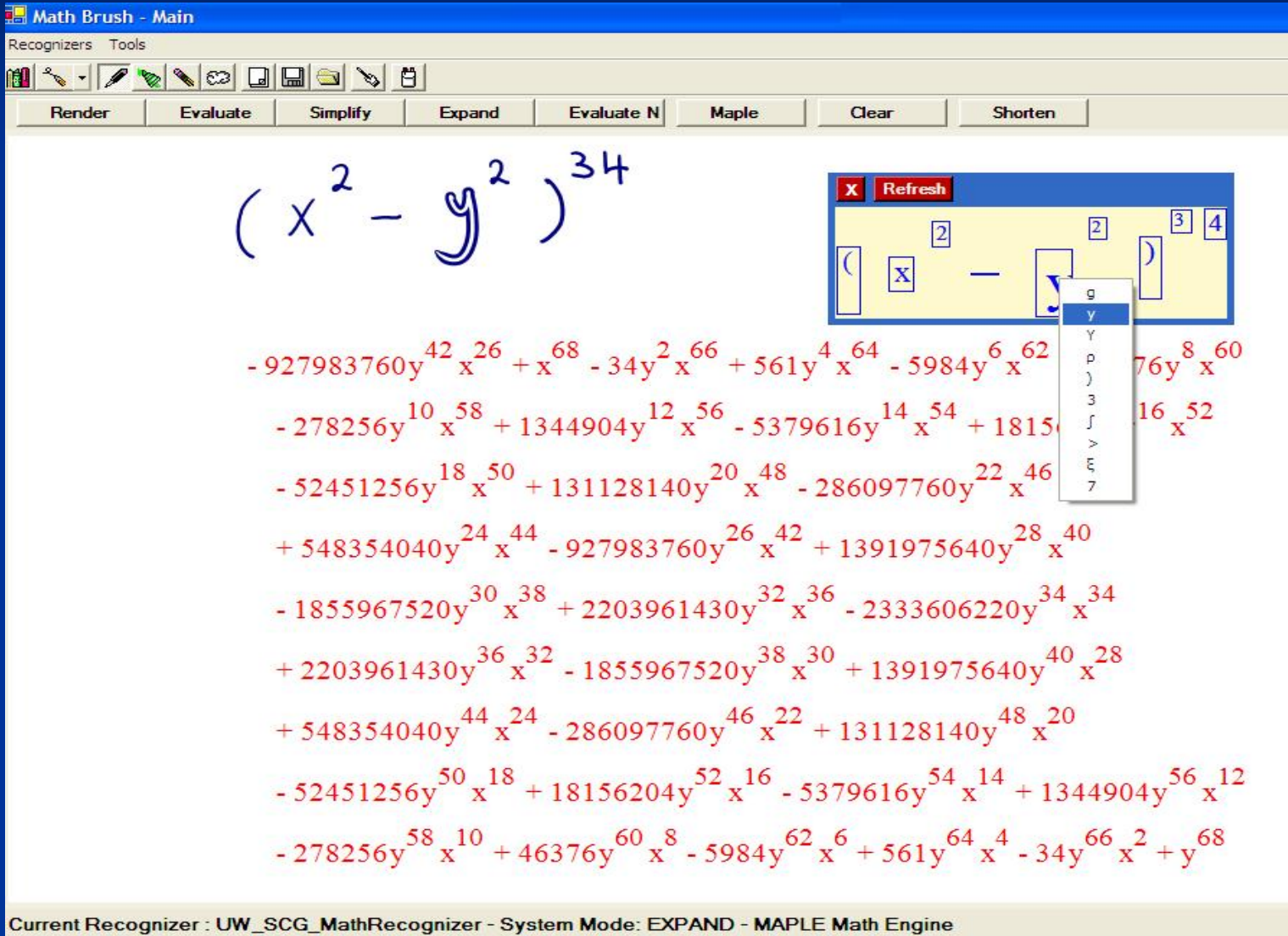
Challenges

- Mathematical symbols appear with wide variation in size and do not necessarily follow baselines
Ex. superscripts and subscripts
- Correct symbols recognition does not lead to a unique math expression.
Ex. $u(x+y)$ u times $x+y$ or u applied to argument $x+y$?
- Rendering two-dimensional math expressions with line-breaking and need for interactivity

MathBrush Components



MathBrush Components User Interface



The screenshot shows the MathBrush software interface. At the top, there is a title bar "Math Brush - Main" and a menu bar with "Recognizers" and "Tools". Below the menu bar is a toolbar with various icons for editing and rendering. A row of buttons includes "Render", "Evaluate", "Simplify", "Expand", "Evaluate N", "Maple", "Clear", and "Shorten".

The main workspace contains a handwritten expression $(x^2 - y^2)^{34}$ in blue ink. To the right of this expression is a small inset window titled "Refresh" showing a grid of mathematical symbols: a left parenthesis, a box containing 'x', a minus sign, a box containing 'y', a right parenthesis, a box containing '2', a box containing '3', and a box containing '4'. A dropdown menu is open over the 'y' box, listing the characters 'g', 'y', 'p', ')', '3', 'j', 'v', 'E', and '7'.

Below the handwritten expression is the expanded polynomial form of $(x^2 - y^2)^{34}$, displayed in red text. The expansion is a sum of terms with various powers of x and y , such as $-927983760y^{42}x^{26} + x^{68} - 34y^2x^{66} + 561y^4x^{64} - 5984y^6x^{62}$ and so on, ending with $-278256y^{58}x^{10} + 46376y^{60}x^8 - 5984y^{62}x^6 + 561y^{64}x^4 - 34y^{66}x^2 + y^{68}$.

At the bottom of the interface, a status bar reads: "Current Recognizer : UW_SCG_MathRecognizer - System Mode: EXPAND - MAPLE Math Engine".

MathBrush Components

Character Recognizer

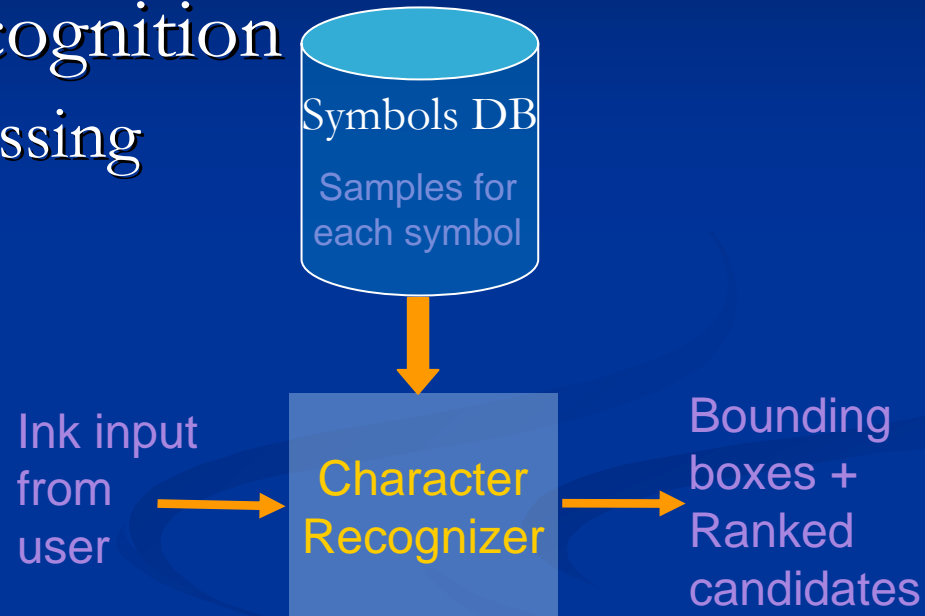
■ Phases of Character Recognition

1. Input (Strokes) Preprocessing

- Stroke joining
- Re-sampling
- Trimming
- Smoothing
- Normalization

2. Segmentation

- Heuristic Approach
Combine hints to choose the number of strokes in the next symbol
- Feature Extraction Approach
Compare features and use **Confusion Matrix** to resolve conflicts



MathBrush Components

Character Recognizer

■ Phases of Character Recognition

1. Input (Strokes) Preprocessing
2. Segmentation
3. Recognition

To produce final confidence the results of the following algorithms are combined

- Elastic Matching
- Deformable Template Matching
- Structural Chain Code Matching

MathBrush Components

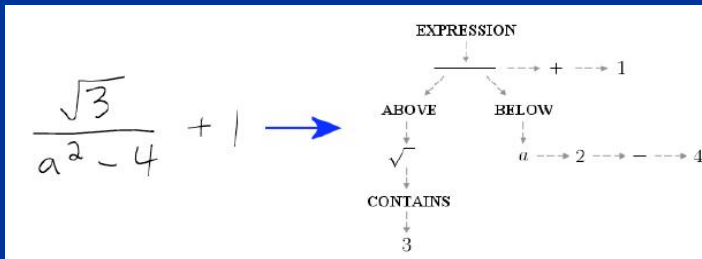
Structural Analyzer

■ Phases of Structural Analysis

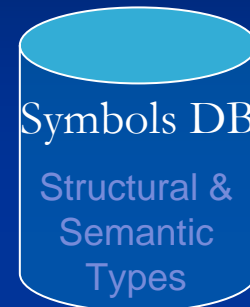
1. Determine Layout

Build an initial baseline tree

Ex.



Bounding boxes + Ranked candidates



Structural Analyzer

Presentation MathML

2. Pre-Parsing

Refine candidates using expected mathematical content

Ex. matching brackets, matching Integral and dx, Numbers ..

MathBrush Components

Structural Analyzer

- Phases of Structural Analysis

1. Determine Layout

2. Pre-Parsing

3. Structural Grouping

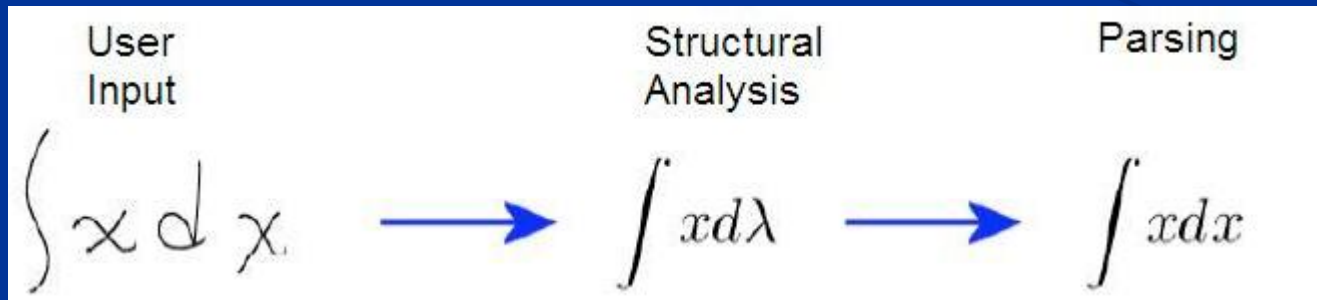
- Use DB information to generate **structural confidence**

- Decide on the best candidate using character and structural confidence

4. Parsing

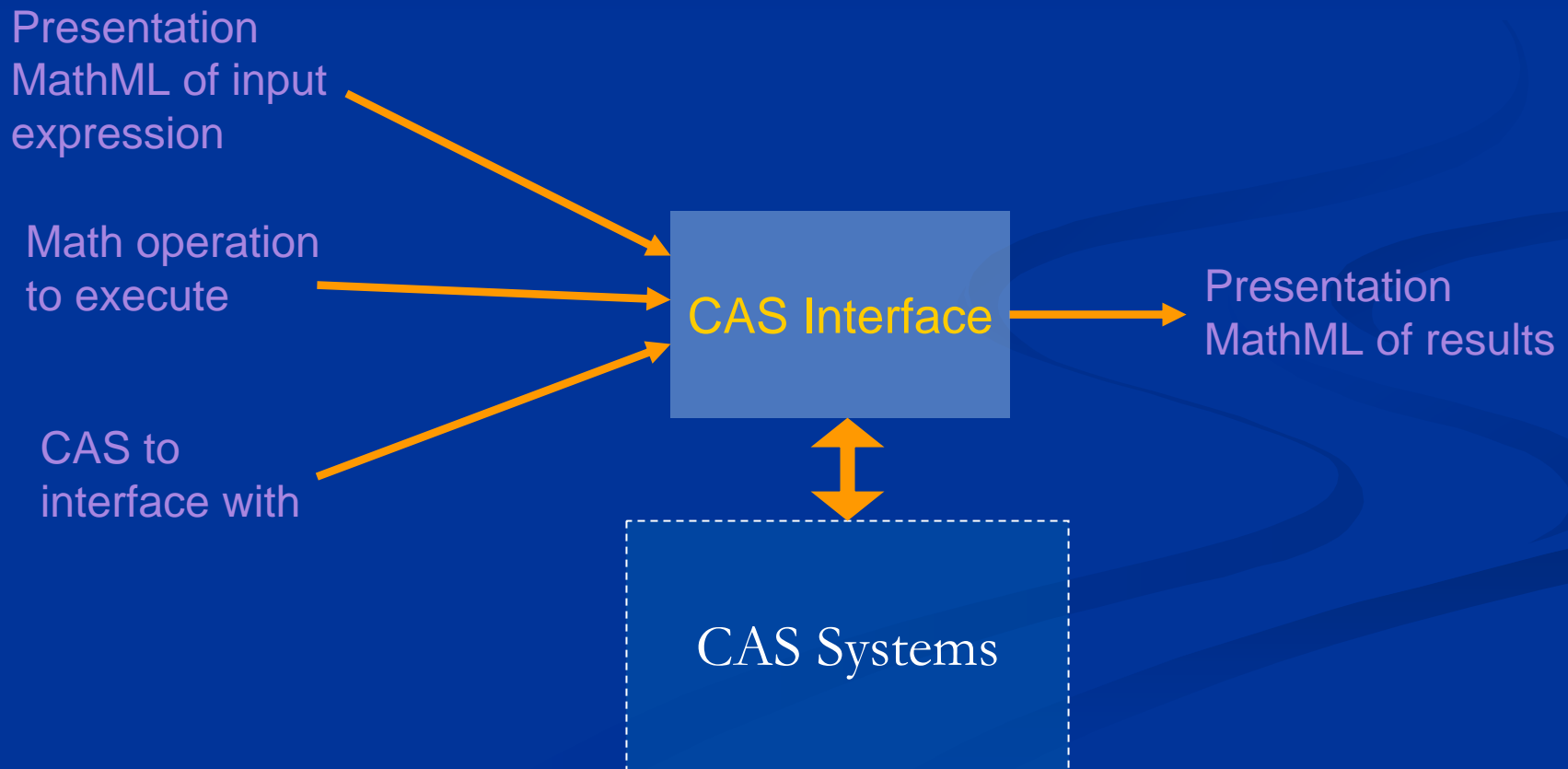
- Consult a database of likely expressions to refine the result

Ex.



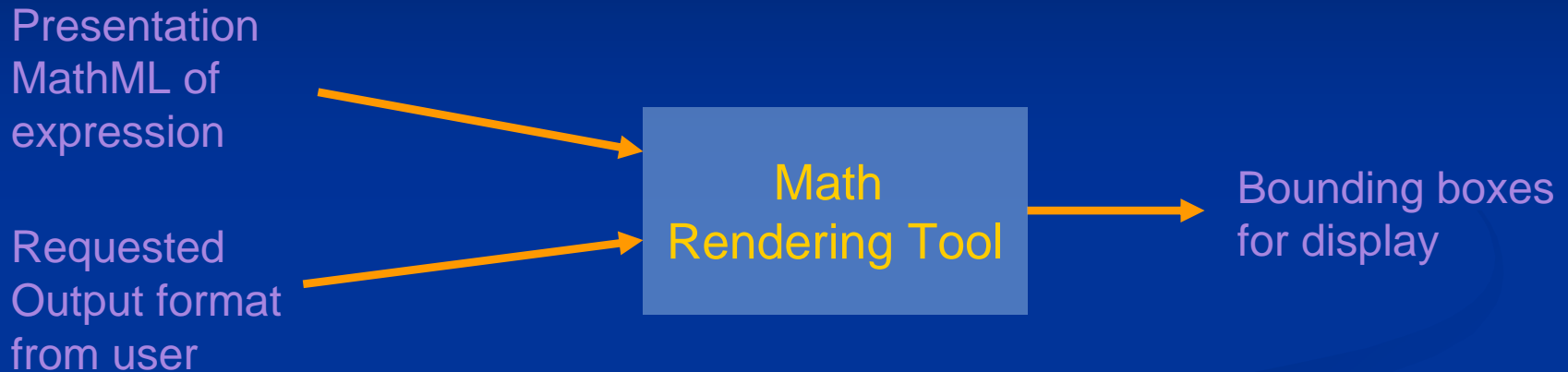
MathBrush Components

CAS Interface Tool



MathBrush Components

Math Rendering Tool



■ Main Features

- Rendering Presentation MathML following W3C recommendations
- Line breaking
- Output format follows user specifications
- Rendering short form for math expressions

Summary

- Objective
 - Builds a pen-based math system while using the power of CAS.

- Current Status
 - MathBrush a preliminary testbed for testing different modules, investigating key issues and getting feedback !!

- Main Features
 - Modular system components
 - Presentation MathML
 - Interaction between different platforms (C#, C++, and Java)

Future Extensions

- Allow the use of more gestures
- Add dynamic context menus
- Allow for more interaction with the results
- Support new classes of expressions: limits, matrices ..
- Use guided input for better character recognition
- Personalize the character recognizer database
- Train the structural analyzer parameters
- Interface with more math systems

DEMO